Phyx 320
Modern Physics

February 5, 2021

Reading: 36.9 - 36.10

Homework #3 and Reading Reflection Due Next Tuesday 11:59 pm

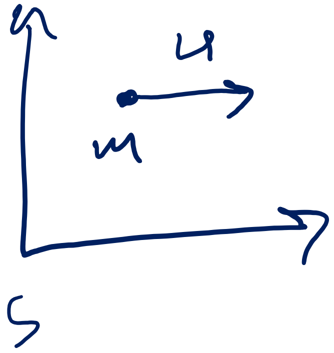
Relativistic Momentum

Derived relativistic momentum

Showed that nothing can travel faster than the speed of light

Speed of light also speed of causality

$$p = \gamma_p \, m \, u$$



$$\gamma_p = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Relativistic Energy

Last quantity to alter, energy

Let's review kinetic energy in Newtonian Mechanics

Newtonian:

$$K = \frac{1}{2} m u^2 , \quad p = mu$$

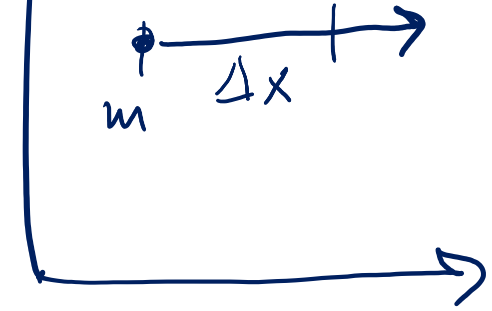
$$= \frac{p^2}{2m}$$ ← different in relativity

Relativistic Energy

We would like to have energy be Lorentz invariant (same in all reference frames)

Let's look at the space-time interval again

$$u = \frac{\Delta x}{\Delta t}$$



S

$$s^2 = c^2 \Delta t^2 - \Delta x^2$$

invariant

$$\left(\frac{m s}{\Delta \tau}\right)^2 = \left(\frac{m}{\Delta \tau}\right)^2 \left[c^2 \Delta t^2 - \Delta x^2\right]$$

$$= (mc)^2 \left(\frac{\Delta t}{\Delta \tau}\right)^2 - \left(m \frac{\Delta x}{\Delta \tau}\right)^2$$

$$p = m \frac{\Delta x}{\Delta \tau} = \gamma_p m u$$

$$= (mc)^2 \left(\frac{\Delta t}{\Delta \tau}\right)^2 - p^2$$

Relativistic Energy

$$\left(\frac{ms}{\Delta\tau}\right)^2 = (mc)^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - p^2$$

invariant

$$\Delta t = \gamma_p \Delta\tau$$

$$= (\gamma_p mc)^2 - p^2$$

Rest Frame: $p = 0$, $\gamma_p \to 1$

$$\left(\frac{ms}{\Delta\tau}\right)^2 = (mc)^2$$

$$(mc)^2 = (\gamma_p mc)^2 - p^2$$

Relativistic Energy

$$\left[(mc)^2 = (\gamma_p mc)^2 - p^2\right] c^2 \qquad \frac{1}{\sqrt{1-x}} = (1-x)^{-1/2}$$

$$(mc^2)^2 = (\gamma_p m c^2)^2 - (pc)^2 \qquad \simeq 1 - (-1/2)x$$

binomial

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1-(u/c)^2}} \simeq mc^2\left(1 + \frac{1}{2}\left(\frac{u}{c}\right)^2\right)$$

$$= mc^2 + \frac{1}{2}mu^2$$

total Energy = E

rest energy

Kinetic Energy

Rest and Kinetic Energy

Two terms contribute to energy:

- Rest energy = energy in rest frame of particle
- Kinetic energy = energy from motion of particle

$$(mc^2)^2 = E^2 - (pc)^2$$

$$E^2 = p^2c^2 + m^2c^4$$

Kinetic Energy

rest energy

$$E \sim K + E_0 + U$$

P - frame dependent

=> E - frame dependent

mc^2 - invariant

Mass-Energy Equivalence

Most famous equation in physics

Any particle with mass has energy no matter it’s motion

Mass is not conserved

- It can become other types of energy and vice versa

$E_0 = mc^2$

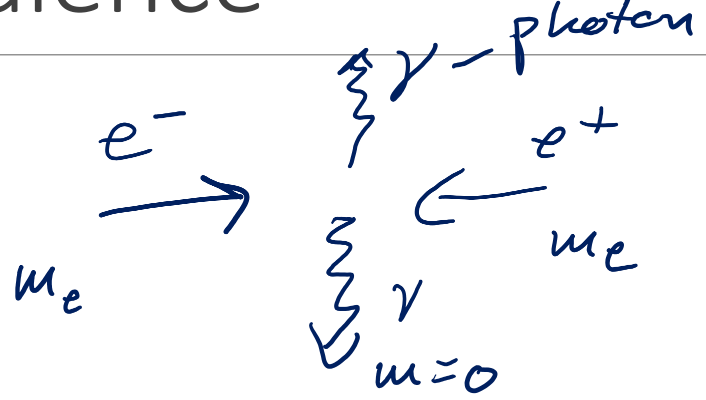
$$E^2 = p^2c^2 + m^2c^4$$

mass changes

Mass-Energy Equivalence

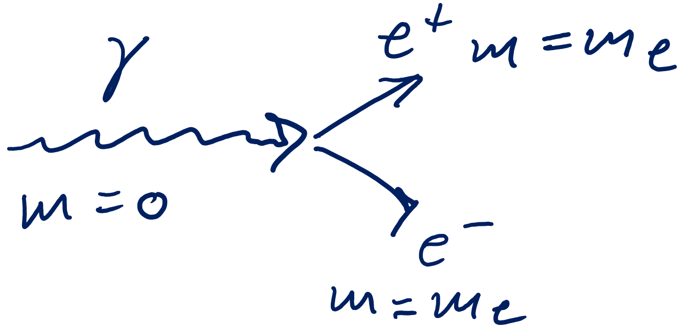
Where can we observe this?

- Particle-antiparticle annihilation
- Pair production
- Nuclear decays



$E_i = 2m_e + K$

$E_f = E_{photon}$



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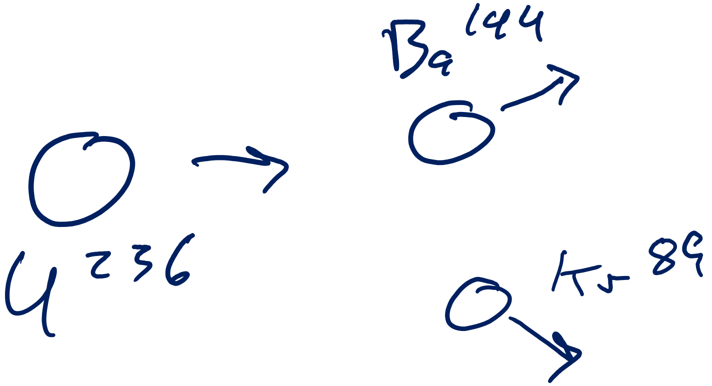


$U^{236} \to Ba^{144} + Kr^{89} + 3n$

$\Delta m = 0.185\,\text{amu} \to K$

Relativity Conclusion

Asserted that all laws of physics are independent of reference frame

- Implies speed of light is constant

Required us to change our understanding of space and time to be frame dependent

Following this through made us change our definitions of momentum and energy

Makes it impossible to accelerate any object faster than the speed of light

Concluded that there's an energy associated with mass

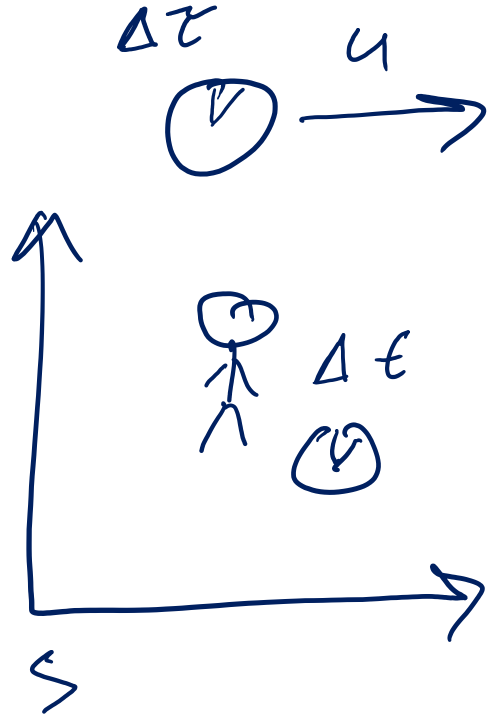
$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$p = \gamma_p\, m\, u$$

$$E = mc^2$$

Homework Questions



$$\Delta t - \Delta\tau = 1.0\,\mu s$$

$$\Delta t = 1.0\ \text{day}$$

$$\Delta t = \sqrt{1-(u/c)^2}\,\Delta\tau$$

$$\Delta\tau = \frac{1}{\sqrt{1-(u/c)^2}}\,\Delta t$$

$$\Delta t - \frac{1}{\sqrt{1-(u/c)^2}}\,\Delta t = 1.0\,\mu s$$

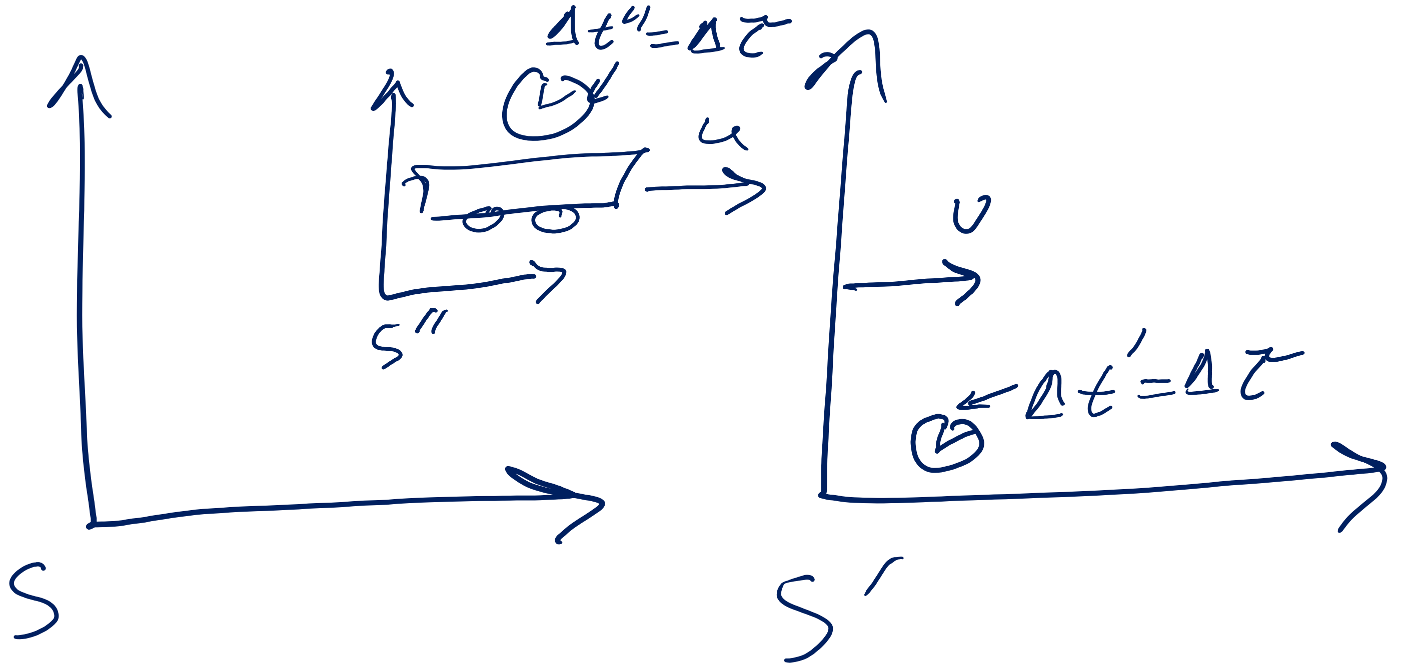
$$\left(1-(u/c)^2\right)^{-1/2} \simeq 1-\left(-\tfrac{1}{2}\right)\left(\frac{u}{c}\right)^2$$

$$(1.0\ \text{day})\left(\frac{1}{\sqrt{1-(u/c)^2}}\right) = 1.0\,\mu s$$

$$= 1 + \frac{1}{2}\left(\frac{u}{c}\right)^2$$

$$(1.0\ \text{day})\left(1+\frac{1}{2}\left(\frac{u}{c}\right)^2\right) = 1.0\,\mu s$$

Homework Questions



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