Phyx 320
Modern Physics

February 1, 2021

Reading: 36.9 - 36.10

Homework #2 and Reading Reflection Due Tuesday 11:59 pm

Lorentz Transformation

Lorentz transformation tells you how space and time change in different frames

$$x' = \gamma(x - vt)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

Derived velocity transformation

$$u' = \frac{u - v}{1 - uv/c^2}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$\beta = \frac{v}{c}$

Relativistic Momentum

We know that the velocity transformation is different in special relativity so what about momentum?

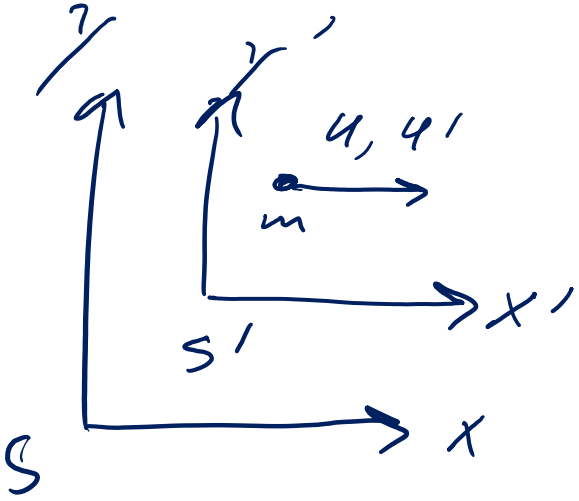
Let's review momentum in Galilean Relativity

Galilean:

$$x' = x - vt$$

$$x = x' + vt$$

$$t' = t$$



$$p = p' + mv$$

$$p = mu$$

$$= m\frac{dx}{dt}$$

$$= m\frac{d}{dt}(x' + vt)$$

$$= m\frac{dx'}{dt} + mv$$

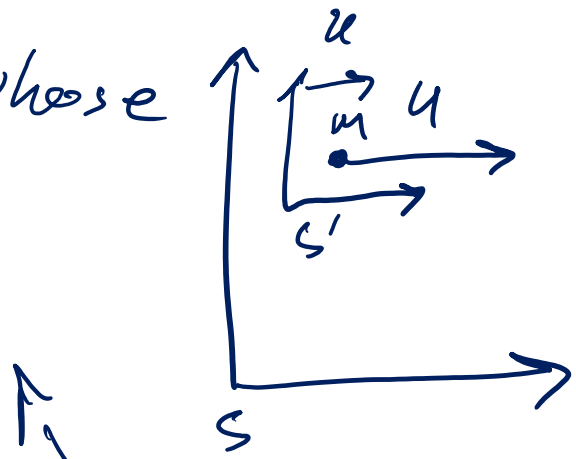
Relativistic Momentum

Now for special relativity whose time should we use?

Everyone can agree on proper time

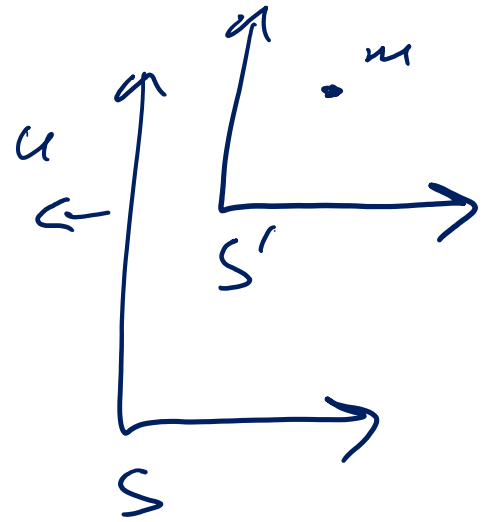
$$p = m\frac{\Delta x}{\Delta t} \quad \Delta t \to \Delta\tau$$

whose



$$\Delta\tau = \sqrt{1-\left(\frac{u}{c}\right)^2}\,\Delta t$$

s' Frame:



$$p = m\frac{\Delta x}{\sqrt{1-\left(\frac{u}{c}\right)^2}\,\Delta t}$$

$$p = m\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}\frac{\Delta x}{\Delta t} \quad \frac{\Delta x}{\Delta t} \leftarrow u$$

$$\gamma_p = \frac{1}{\sqrt{1-(u/c)^2}}$$

$$\boxed{p = m\gamma_p u}$$

Fundamental Speed Limit

Let's try to accelerate a particle faster than light

$$p = \gamma_p m u$$

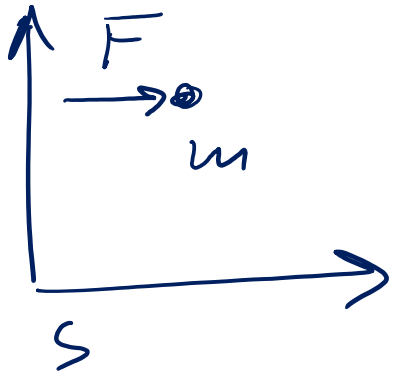
$$Ft = \frac{mu}{\sqrt{1-\left(\frac{u}{c}\right)^2}}$$

$$(Ft)^2\left(1-\left(\frac{u}{c}\right)^2\right) = (mu)^2$$

$$(Ft)^2 = (mu)^2 + \left(\frac{Ftu}{c}\right)^2$$

$$(Ft)^2 = \left[m^2 + \left(\frac{Ft}{c}\right)^2\right]u^2$$

$$\rightarrow u = \frac{Ft}{\sqrt{m^2+\left(\frac{Ft}{c}\right)^2}}$$



$$F = \frac{dp}{dt}$$

$$p = Ft$$

Galileen

$$u = \frac{Ft}{m}$$

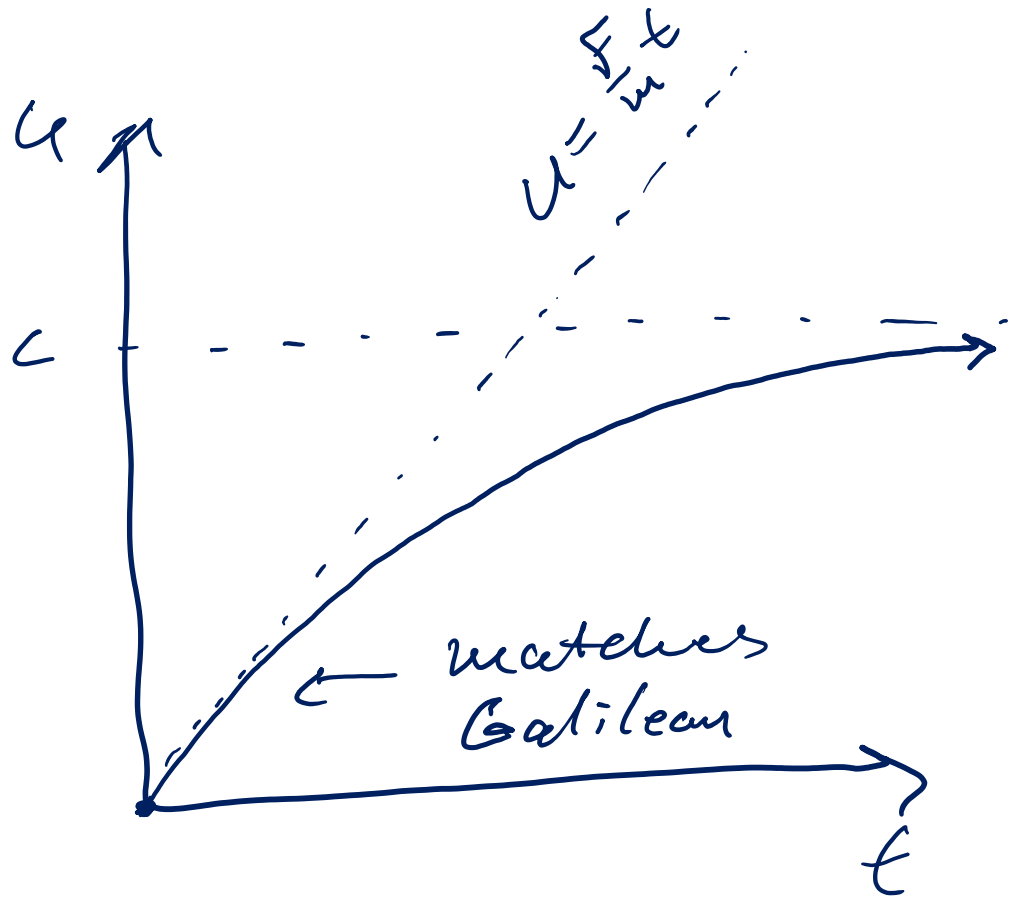
Fundamental Speed Limit

Let's try to accelerate a particle faster than light

$$u = \frac{Ft}{\sqrt{m^2 + \left(\frac{Ft}{c}\right)^2}}$$

$$t = 0, \quad u = 0$$

$$t \to \infty, \quad u \to \frac{Ft}{\sqrt{\left(\frac{Ft}{c}\right)^2}} = c$$

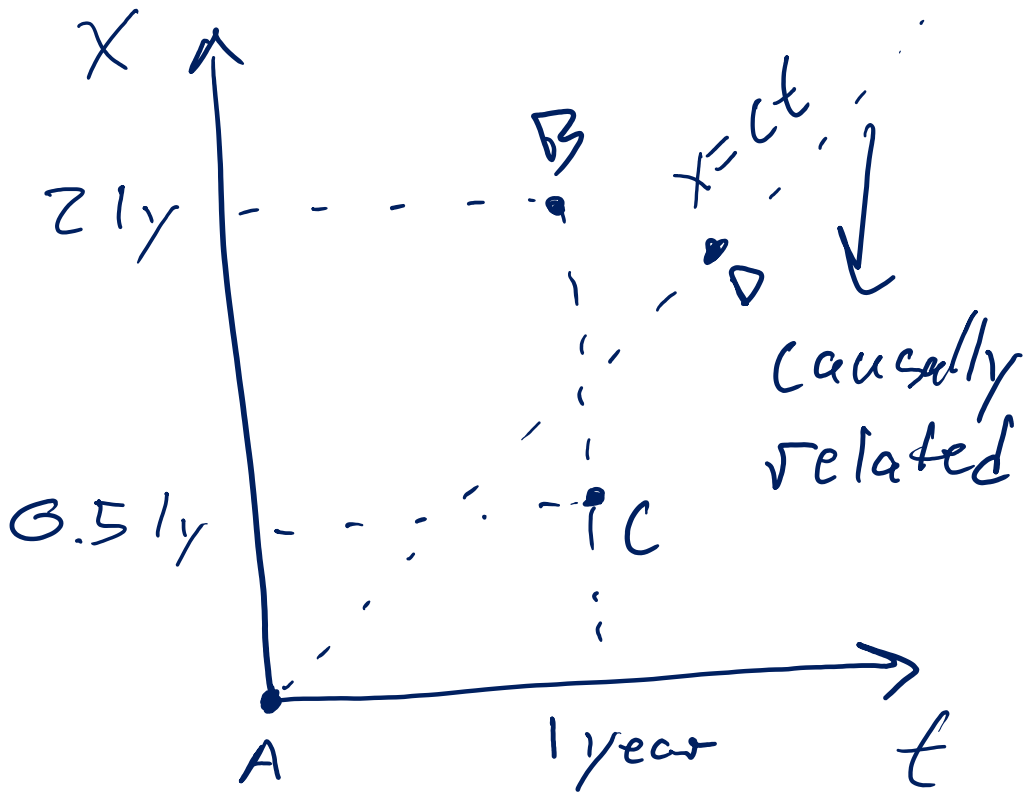


Causality

The speed of light is the limit for any causal influence

Event A and Event B can only be related if information can flow from one event to the next

The speed limit for this information flow is the speed of light (speed of causality)



Causality

Space-time interval can tell you if two events could be causally related

$s^2 \geq 0$ causally related, more time than space

$s^2 < 0$ not causally related, more space than time

$$s^2 = (c\Delta t)^2 - (\Delta x)^2$$

$$s_{AB}^2 = (c\,(1\text{year}))^2 - (0.5\,\text{ly})^2$$

$$= 0.75\ \text{ly}^2$$

Causally related

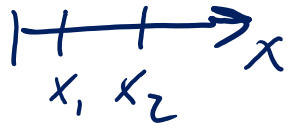
$$s_{AC}^2 = (c\,(1\text{year}))^2 - (2\,\text{ly})^2$$

$$= -3\ \text{ly}^2$$

can't be Causally related

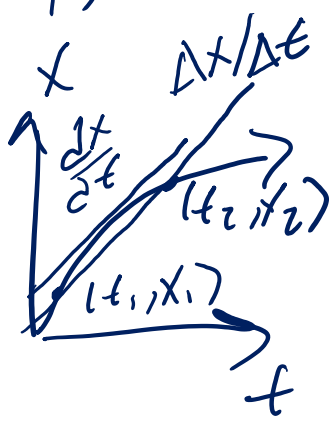
Homework Questions

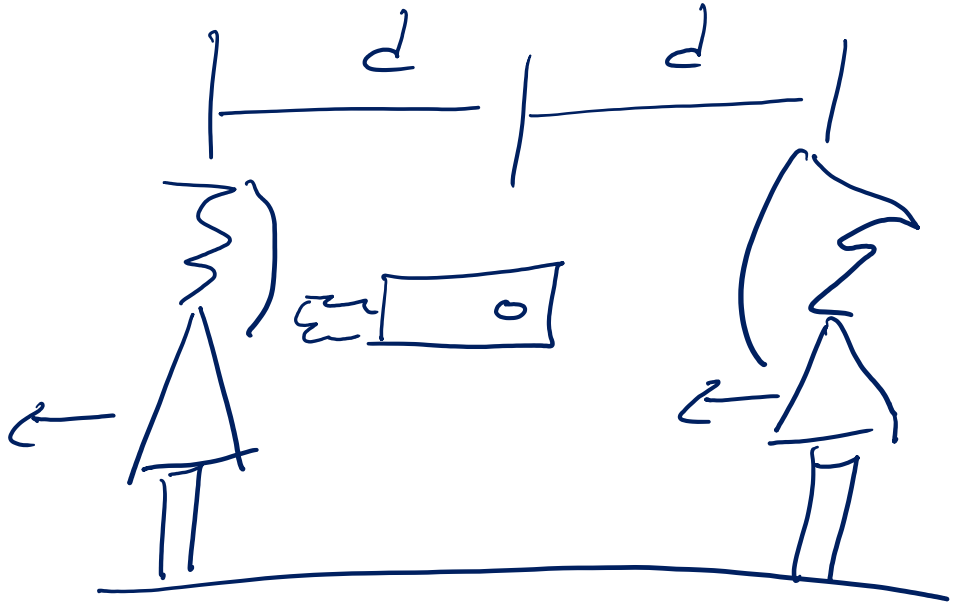
$\Delta x = (x_2 - x_1)$



$dx = \lim_{x_1 \to x_2} (x_2 - x_1)$

$\frac{dx}{dt}$





$t = \frac{d}{c}$

Homework Questions

$d = 4.13 \times 10^{13}$ km

How long would signal take?

1. Spectral
2. Galelian

$t = \frac{d}{c} = \frac{4.13 \times 10^{13}\ \text{km}}{3 \times 10^{8}\ \text{m/s}} \times 10^{3}$ ↓ = N S

$= 4.37$ years $\therefore$ (N S) $\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{hr}}\right)$

$t = \frac{d}{c - 0.2c} = \frac{d}{0.8c} = \frac{4.37}{0.8}$ years $\therefore$ 4.37 $\times \left(\frac{1\ \text{year}}{365\ \text{days}}\right)$

Homework Questions

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