Phyx 320
Modern Physics

January 29, 2021

Reading: 36.5 - 36.8

Homework #2 and Reading Reflection Due Next Tuesday 11:59 pm

Time Dilation

Time depends on reference frame

Simultaneous events in one frame are not necessarily simultaneous in other frames

Proper time = time measured in reference frame of clock

Moving clocks run slow

$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}}$$

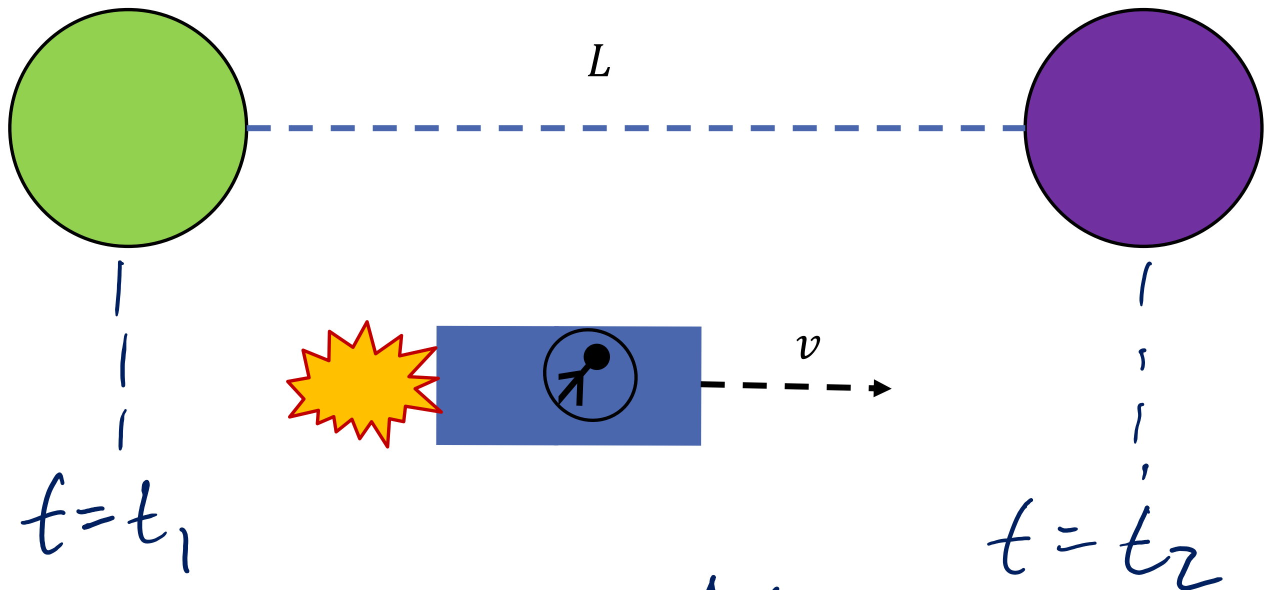
$$\beta = \frac{v}{c}$$

$$\Delta t \geq \Delta\tau$$

Length Contraction

What about distances?

Let's say we have a spaceship that travels from one planet to another at constant velocity



Δx

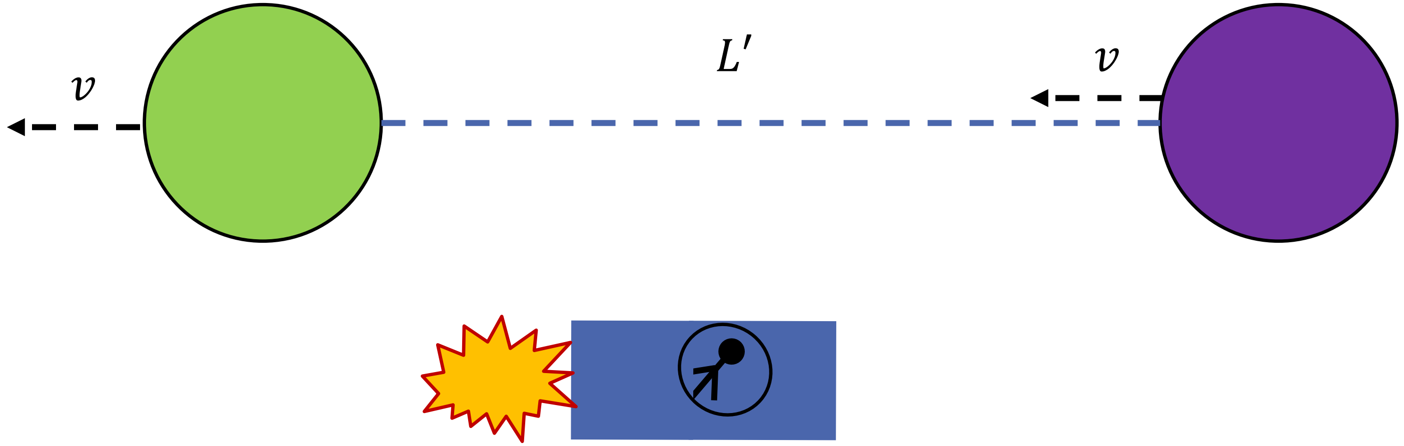
$$V = \frac{L}{\Delta t}$$

Length Contraction

In astronauts frame the planets are moving at constant velocity

Passing each planet takes place at same location in astronaut frame

Time measured between two events that happen at same location = proper time



$$v = \frac{L}{\Delta t}$$

proper time

$$v = \frac{L'}{\Delta t'} = \frac{L}{\Delta t}$$

$$\Delta t = \frac{1}{\sqrt{1-\beta^2}} \Delta \tau$$

$$\frac{L'}{\sqrt{1-\beta^2}\,\Delta t} = \frac{L}{\Delta t}$$

$$\Delta \tau = \sqrt{1-\beta^2}\,\Delta t$$

$$L' = \sqrt{1-\beta^2}\,L$$

Length Contraction

Distances between two objects are frame dependent

Length measured in rest frame of objects = proper length

Moving lengths “shrink”

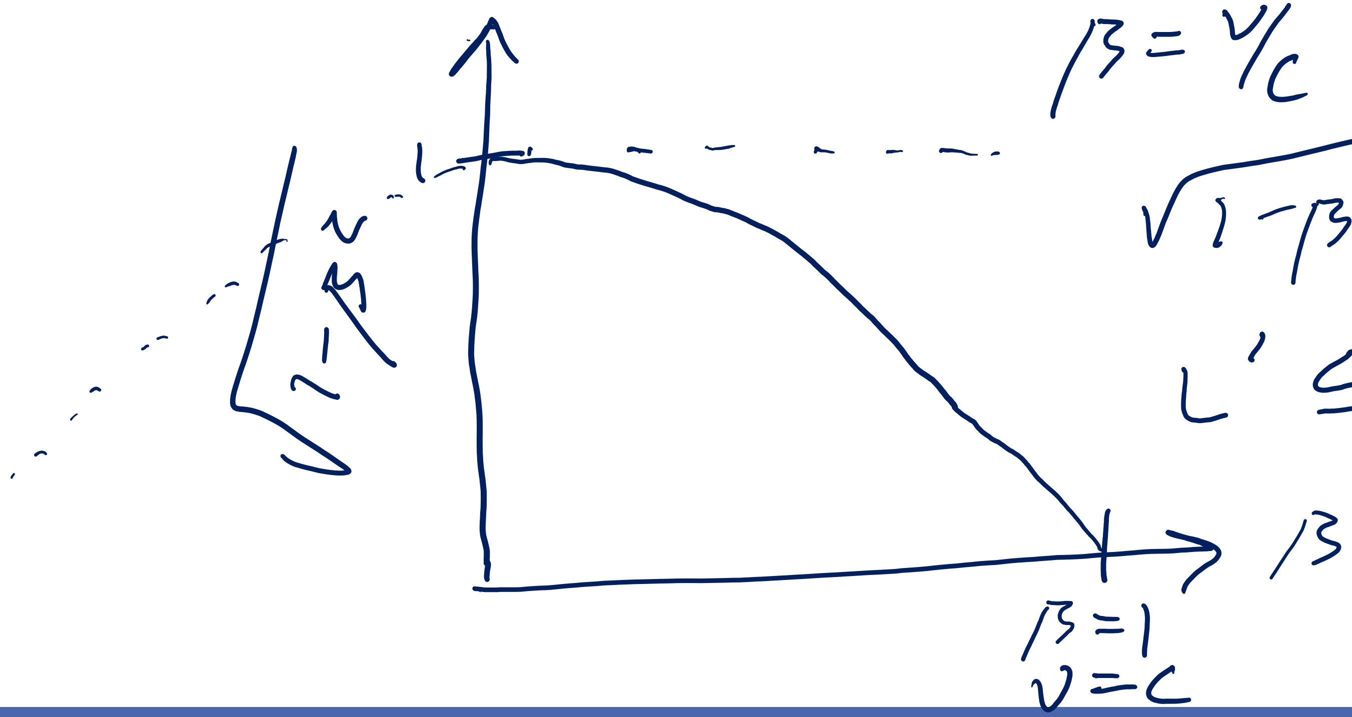
$$\Delta t = \frac{1}{\sqrt{1-\beta^2}} \Delta \tau$$

$$L' = \sqrt{1-\beta^2}\, l$$

$$\beta = v/c$$

$$\sqrt{1-\beta^2} \leq 1$$

$$L' \leq l$$



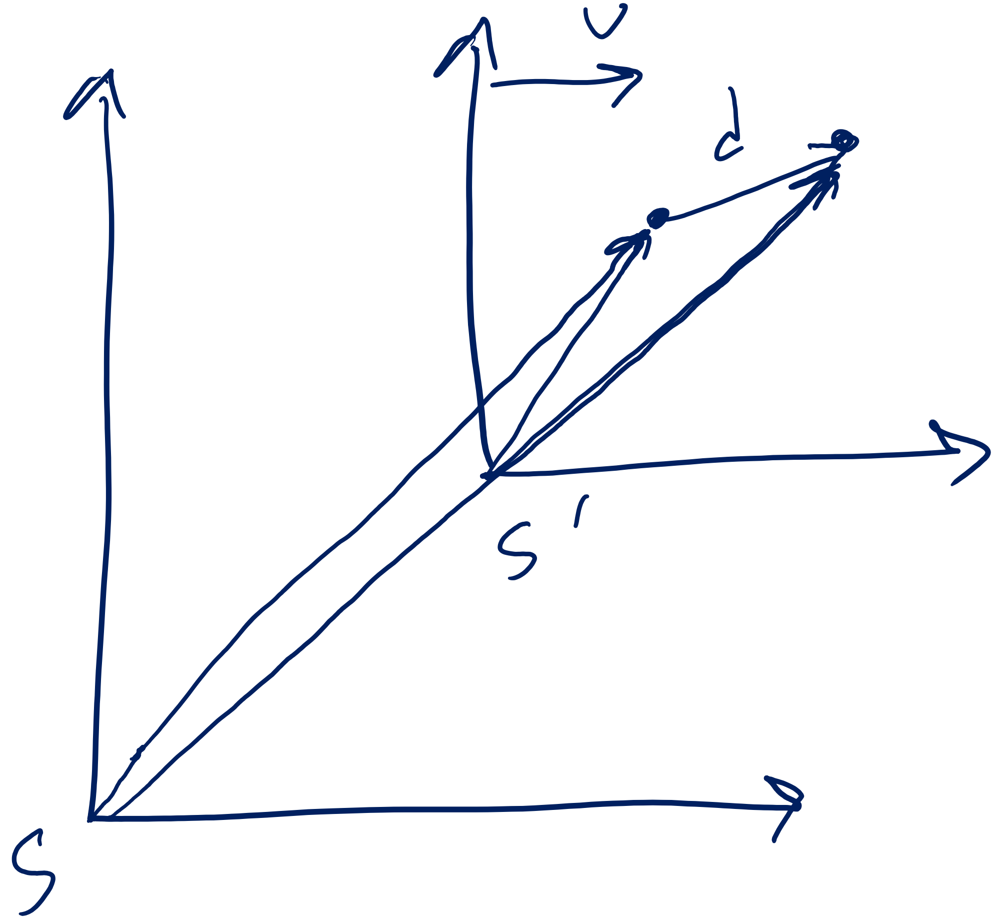
Space-time Interval

In space and time depend on reference frame, is there any space-time quantity which all frames agree?

Invariant quantity = quantity that is independent of frame

In Galilean relativity, distance is invariant

$$d^2 = \Delta x^2 + \Delta y^2 + \Delta z^2$$



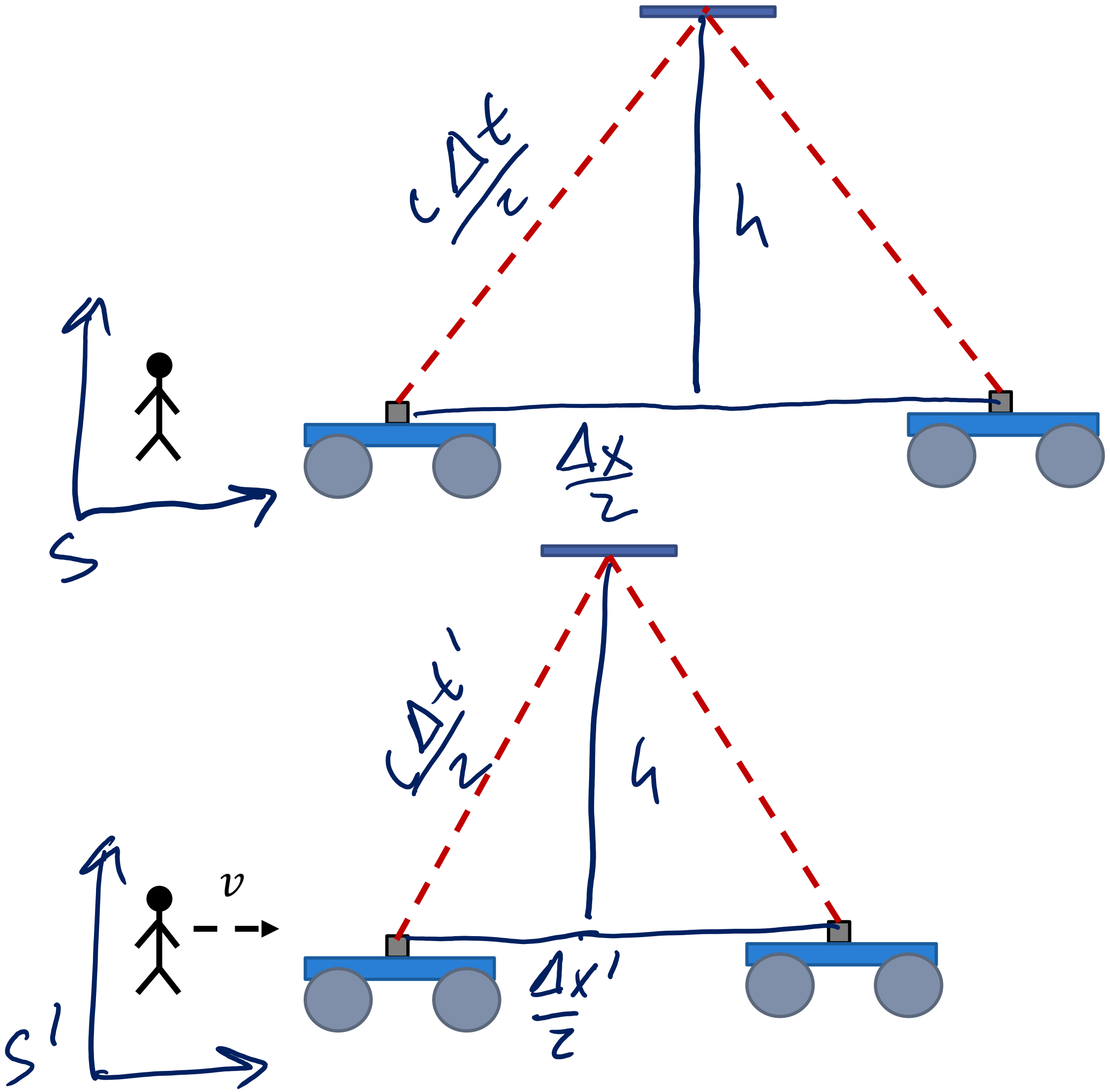
Space-time Interval

Return to light clock but now two different observers

One standing on the ground, the other running at constant velocity

$$h^2 + \left(\frac{\Delta x}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

$$h^2 + \left(\frac{\Delta x'}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$



Space-time Interval

Return to light clock but now two different observers

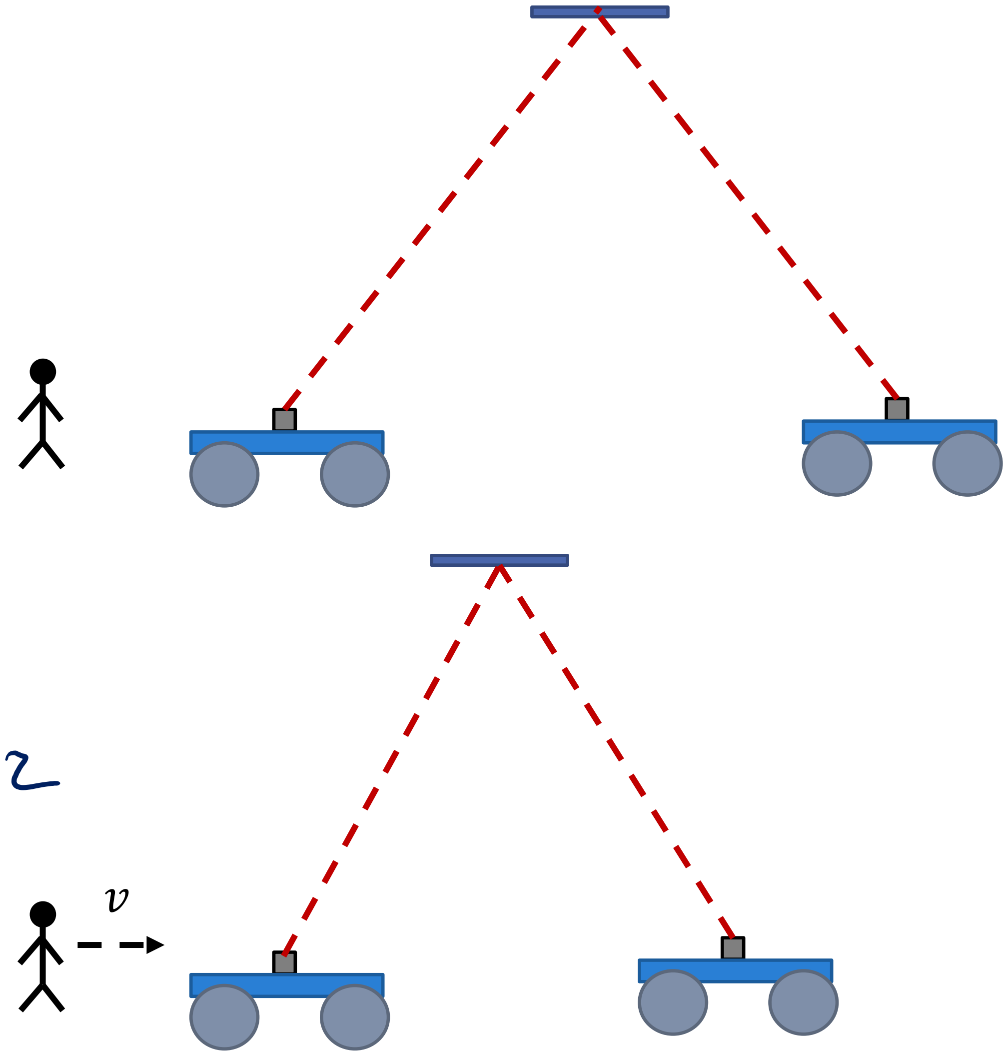
One standing on the ground, the other running at constant velocity

$$h^2 = \left(\frac{c\Delta t}{2}\right)^2 - \left(\frac{\Delta x}{2}\right)^2$$

$$h^2 = \left(\frac{c\Delta t'}{2}\right)^2 - \left(\frac{\Delta x'}{2}\right)^2$$

$$\underline{c^2\Delta t'^2 - \Delta x'^2} = c^2\Delta t^2 - \Delta x^2$$

$$s^2 = c^2\Delta t^2 - \Delta x^2$$



Space-time Interval

Space-time interval is independent of reference frame = invariant

Useful since every observer agrees on it, we can use it to define "distances" between events

$$s^2 = c^2\,\Delta t^2 - \Delta x^2$$

$$s^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$s^2 = c^2 t^2 - \vec{r} \cdot \vec{r}$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Lorentz Transform

Can we make a general transformation that captures special relativity?

We want it to look like Galilean relativity at low velocity

Keep speed of light constant

Transform both space and time

Galilean:

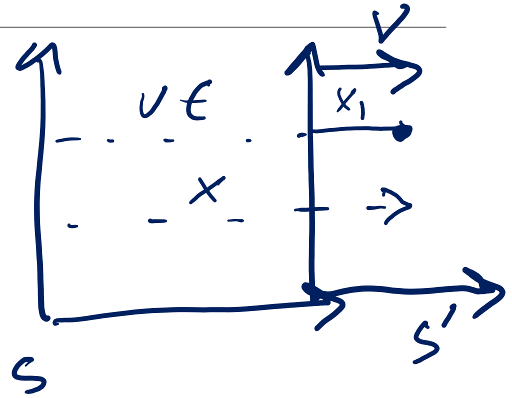
$$x = x' + vt$$

Lorentz:

$$x = \gamma(x' + vt')$$

$$x' = \gamma(x - vt)$$

$$\gamma = 1, \quad v \ll c$$



$$t \neq t'$$

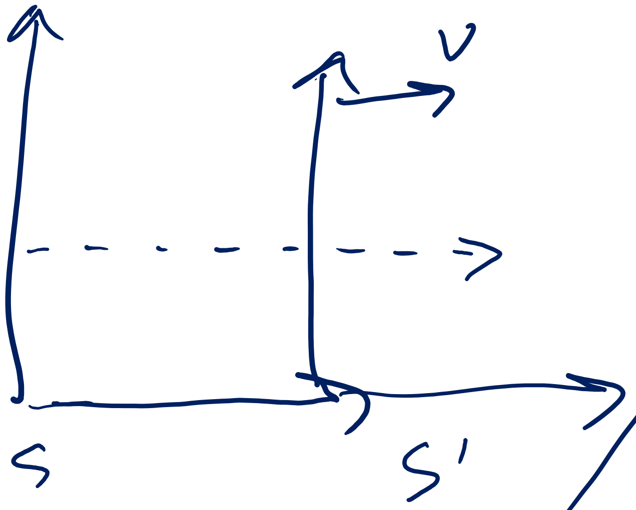
Lorentz Transform

$$x = \gamma(x' + vt')$$

$$x' = \gamma(x - vt)$$

$$ct' = \gamma(ct - vt) \qquad ct = \gamma(ct' + vt')$$

$$t' = \gamma\,\frac{c-v}{c}\,t \qquad \Leftarrow t = \gamma\,\frac{c+v}{c}\,t'$$



$$t' = \gamma\left(\frac{c-v}{c}\right)\gamma\left(\frac{c+v}{c}\right)t' \qquad \beta = v/c$$

$$1 = \gamma^2\left(\frac{c^2 - v^2}{c^2}\right)$$

$$\gamma = \sqrt{\frac{c^2}{c^2 - v^2}} = \sqrt{\frac{1}{1 - (v/c)^2}}$$

$$x = ct$$

$$x' = ct'$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

Lorentz Transform

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$x' = \gamma(x - vt)$$

$$x = \gamma(x' + vt')$$

$$1/\gamma^2 = \frac{1}{\left(\frac{1}{\sqrt{1-\beta^2}}\right)^2}$$

$$= 1-\beta^2 = 1-\left(\frac{v}{c}\right)^2$$

$$\frac{x}{\gamma} = x' + vt'$$

$$\frac{x}{\gamma} - vt' = x'$$

$$\frac{x}{\gamma} - vt' = \gamma(x - vt)$$

$$-vt' = \gamma x - \frac{x}{\gamma} - v\gamma t$$

$$t' = \gamma t - \frac{\gamma - 1/\gamma}{v}\, x$$

$$\boxed{t' = \gamma\left(t - \frac{v}{c^2}x\right)}$$

$$\frac{\gamma - 1/\gamma}{v}$$

$$= \gamma\left(\frac{1 - 1/\gamma^2}{v}\right)$$

$$= \gamma\left(\frac{\cancel{1} - \cancel{1} + \left(\frac{v^2}{c^2}\right)}{\cancel{v}}\right)$$

$$= \gamma\,\frac{v}{c^2}$$

Lorentz Transform

$x' = \gamma(x - vt)$

$y' = y$

$z' = z$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

$x = \gamma(x' + vt')$

$y = y'$

$z = z'$

$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$

$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$

unitless

$v = 0$

$\gamma = 1$

$v \to 0$ limit:

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

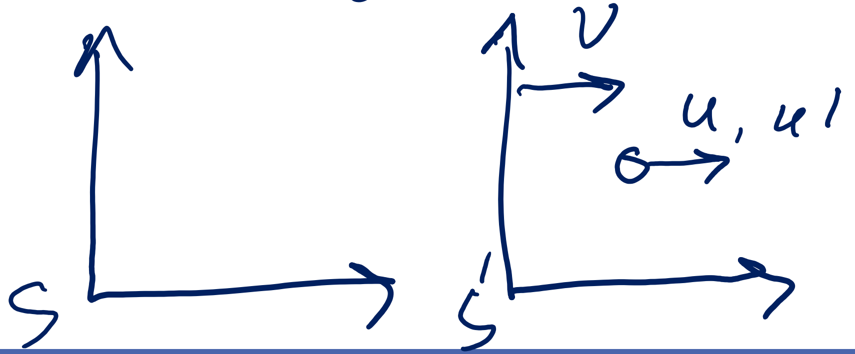
$t' = t$

Relativistic Velocity

Now that we have our transformation what do velocities look like?

$$u' = \frac{dx'}{dt'}$$

$$u = \frac{dx}{dt}$$



$$u' = \frac{dx'}{dt'}$$

$$= \frac{d\left(\gamma(x - vt)\right)}{d\left(\gamma\left(t - \frac{v}{c^2}x\right)\right)}$$

$$= \frac{\cancel{\gamma}(dx - v\,dt)}{\cancel{\gamma}\left(dt - \frac{v}{c^2}dx\right)}$$

$$= \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u - v}{1 - \frac{v}{c^2}u}$$

Relativistic Velocity

$$\boxed{u' = \frac{u - v}{1 - \frac{uv}{c^2}}}$$

$$u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$

$v \ll c$

$1 - \frac{uv}{c^2} \rightarrow 1$

$u' = u - v$

$E^2 = p^2c^2 + m^2c^4$

Relativistic Velocity