Phyx 320
Modern Physics

April 26, 2021

Reading: 42.5-42.7

Homework #12 Due Next Tuesday

Radiation

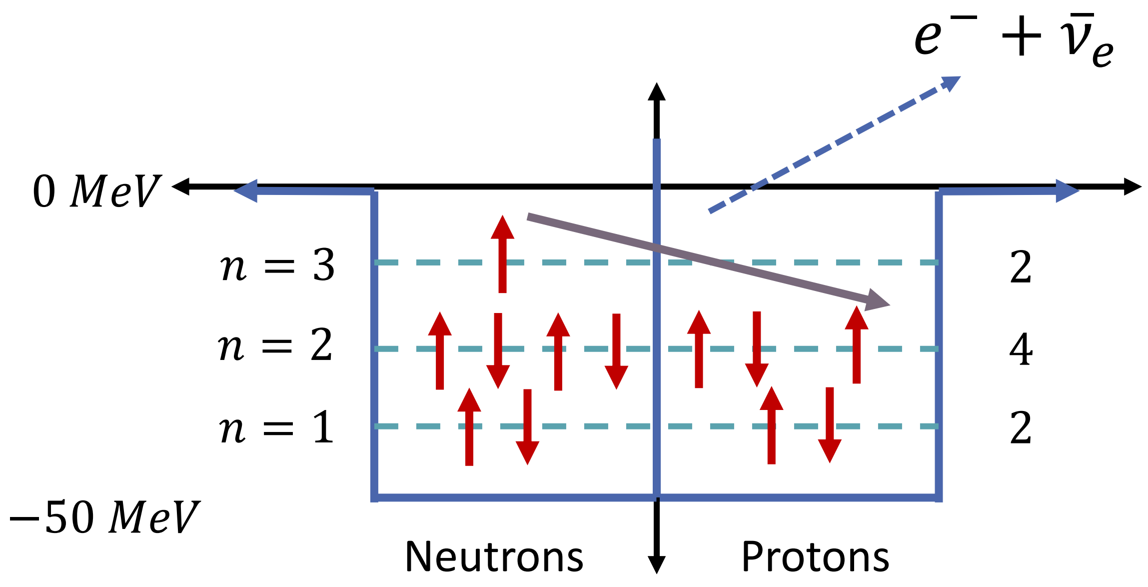
When nuclei transition from a higher energy state to a lower one, they can emit three types of radiation:

- Alpha – ^{4}He nucleus, 2 protons and 2 neutrons, +2e charge, stopped by a sheet of paper
- Beta – electron (or positron), -e charge, stopped by a few mm of aluminum
- Gamma – high-energy photon, 0 charge, stopped many cm of lead

This process is called radioactive decay

For example, boron-12 beta decays to carbon-12, emission of electron is needed to conserve charge

$$^{12}B \rightarrow {}^{12}C + e^{-} + \bar{\nu}_e$$



Nuclear Decay

Nuclear decay is a random process so we can only know the probability for a given nucleus to decay

$$\text{Prob(decay in } \Delta t) = r \Delta t$$

(r: rate)

$$\text{Number of decays} = N r \Delta t$$

(N: # of nuclei)

$$\Delta N = -N r \Delta t$$

$$\frac{\Delta N}{\Delta t} = -N r$$

$$\frac{dN}{dt} = -N r$$

$$\frac{dN}{N} = -r\,dt$$

$$\int_{N_0}^{N} \frac{dN'}{N'} = \int_0^t -r\,dt'$$

$$\ln N - \ln N_0 = -r t$$

$$\ln \frac{N}{N_0} = -r t$$

$$N = N_0 e^{-rt}$$

Nuclear Decay

Defining the lifetime as:

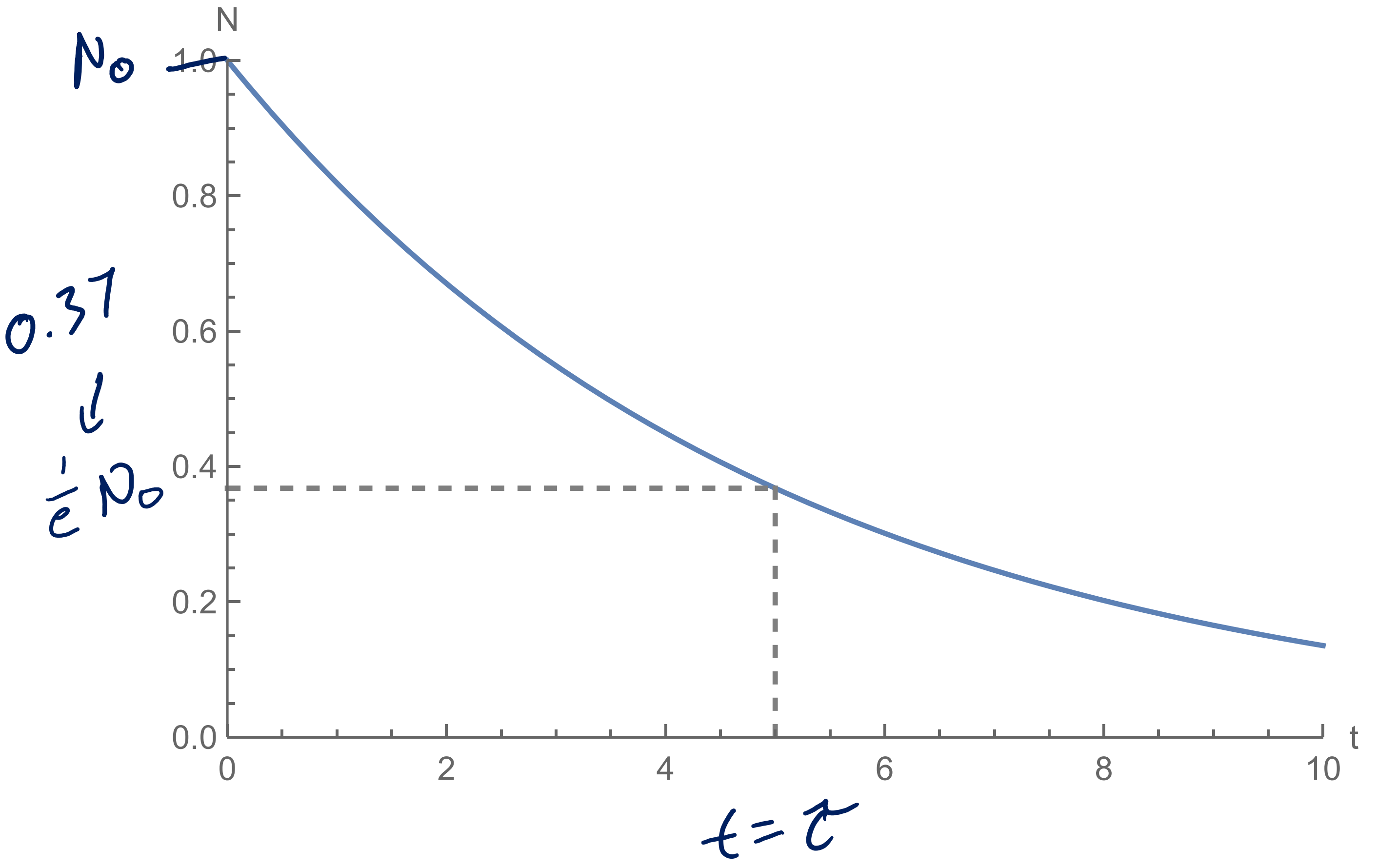
$$\tau = \frac{1}{r}$$

The number of nuclei at some time is:

$$N = N_0 e^{-\frac{t}{\tau}}$$

A sample of radioactive material decays exponentially with time

Lifetime depends on the probability of a decay and is independent of starting number of nuclei



Half-life

It's more convenient to reparametrize in terms of time it takes for number to decrease by half

$$N = N_0 e^{-t/\tau}$$

$$\frac{N_0}{2} = N_0 e^{-t_{1/2}/\tau}$$

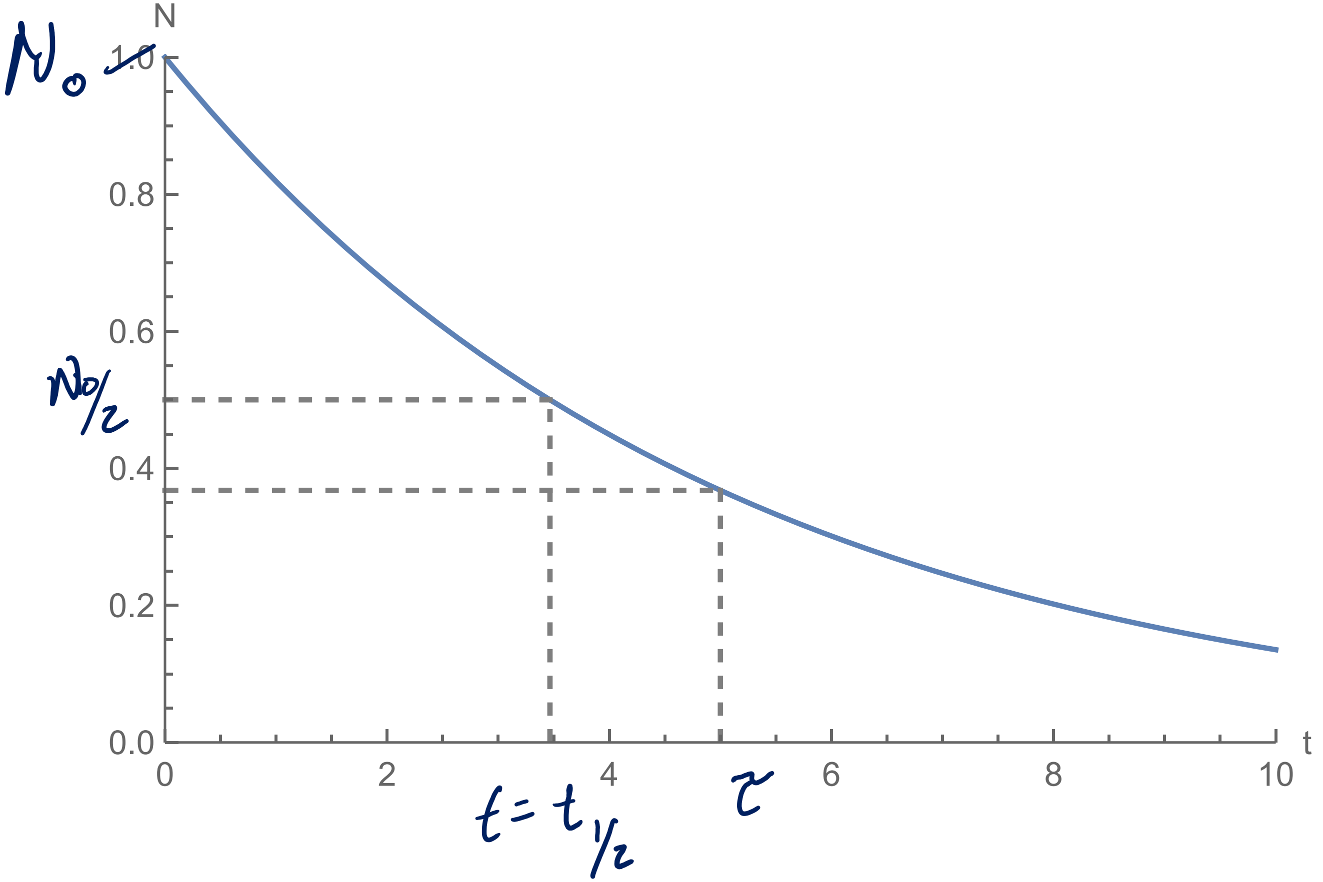
$$2^{-1}$$

$$\ln\left(\frac{1}{2}\right) = -t_{1/2}/\tau$$

$$-\ln 2 = -t_{1/2}/\tau$$

$$t_{1/2} = \ln 2 \; \tau$$

$\uparrow$ 0.69



Half-life

For unstable nuclei, after one half-life half the nuclei will turn into another element called daughter nuclei

Each radioactive isotope has its own half-life (^{14}C: 5,730 years, ^{218}Pb: 15 s)

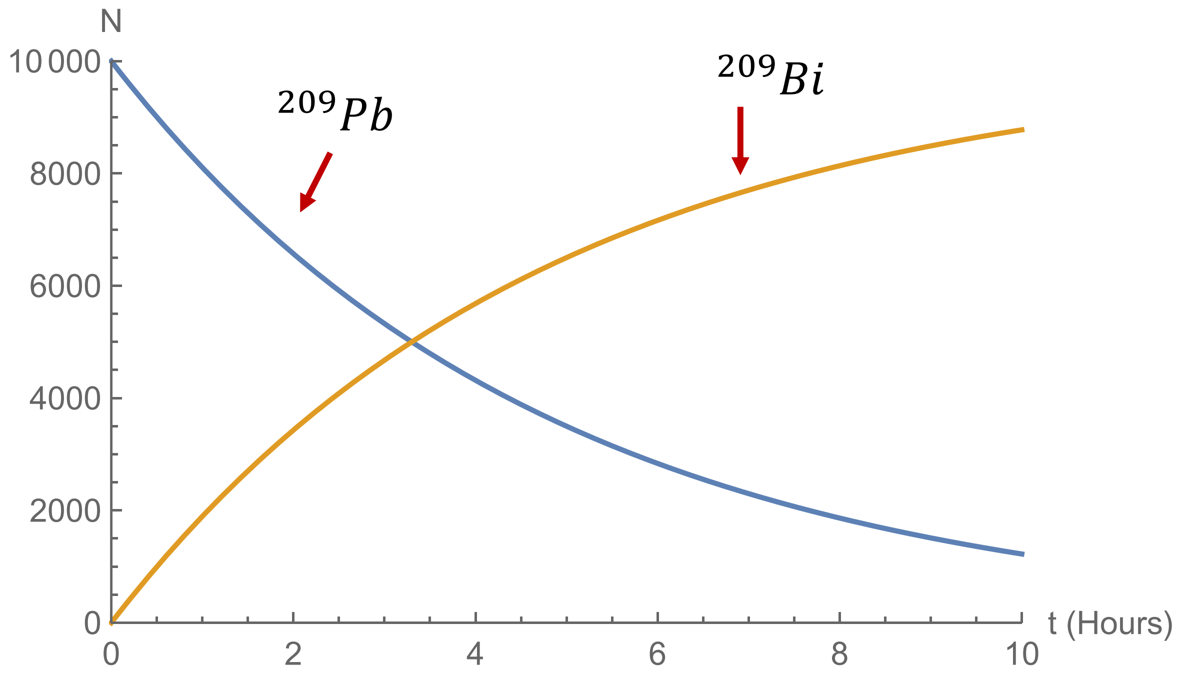
We can't know which nuclei will decay since the process is random

For example, if we have a sample of 10,000 ^{209}Pb nuclei after the half-life of 3.3 hours, we'd have 5,000 ^{209}Pb nuclei and 5,000 ^{209}Bi nuclei

After 6.6 hours, we'd have 2,500 ^{209}Pb nuclei and 7,500 ^{209}Bi nuclei

$$^{209}Pb \rightarrow \, ^{209}Bi + e^- + \bar{\nu}_e$$

$$t_{\frac{1}{2}} = 3.3\ h$$



Radioactive Dating

Radioactive decay allows a variety of dating techniques

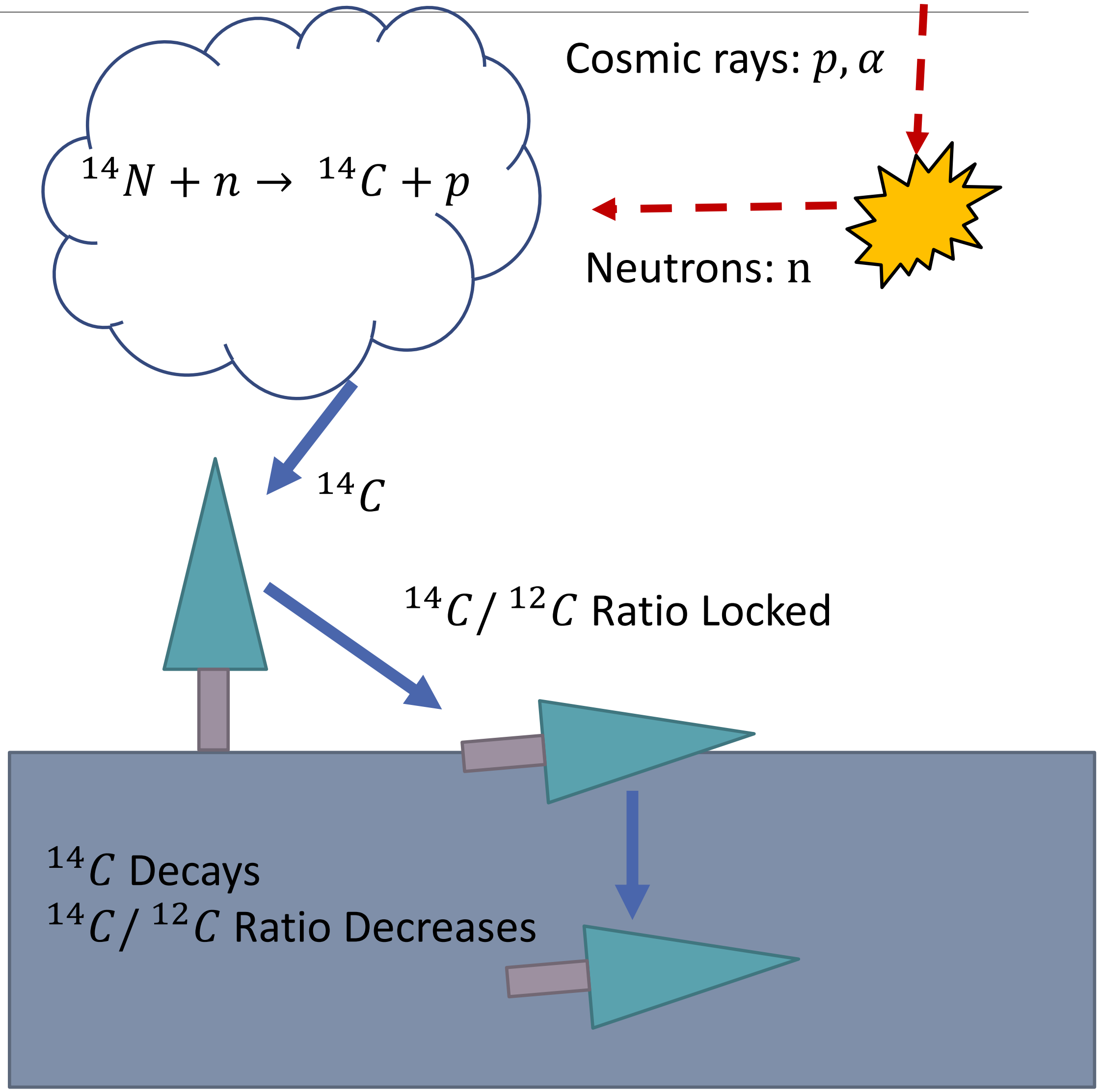
We have to use ratios of unstable and stable nuclei since we never know the initial amount (N_0)

Carbon dating takes the ratio of $^{14}C/^{12}C$

^{14}C has a half-life of 5,730 years so it can't be found in geological samples

But cosmic rays create ^{14}C and all living things exchange carbon with atmospheric carbon dioxide which has a $^{14}C/^{12}C$ ratio of 1.3×10^{-12}

Once something dies and is buried it no longer exchanges with the atmosphere and the ratio decreases due to ^{14}C decays



Nuclear Fission

Like we saw with cosmic rays in the atmosphere, nuclei can undergo reactions when struck with incoming particles

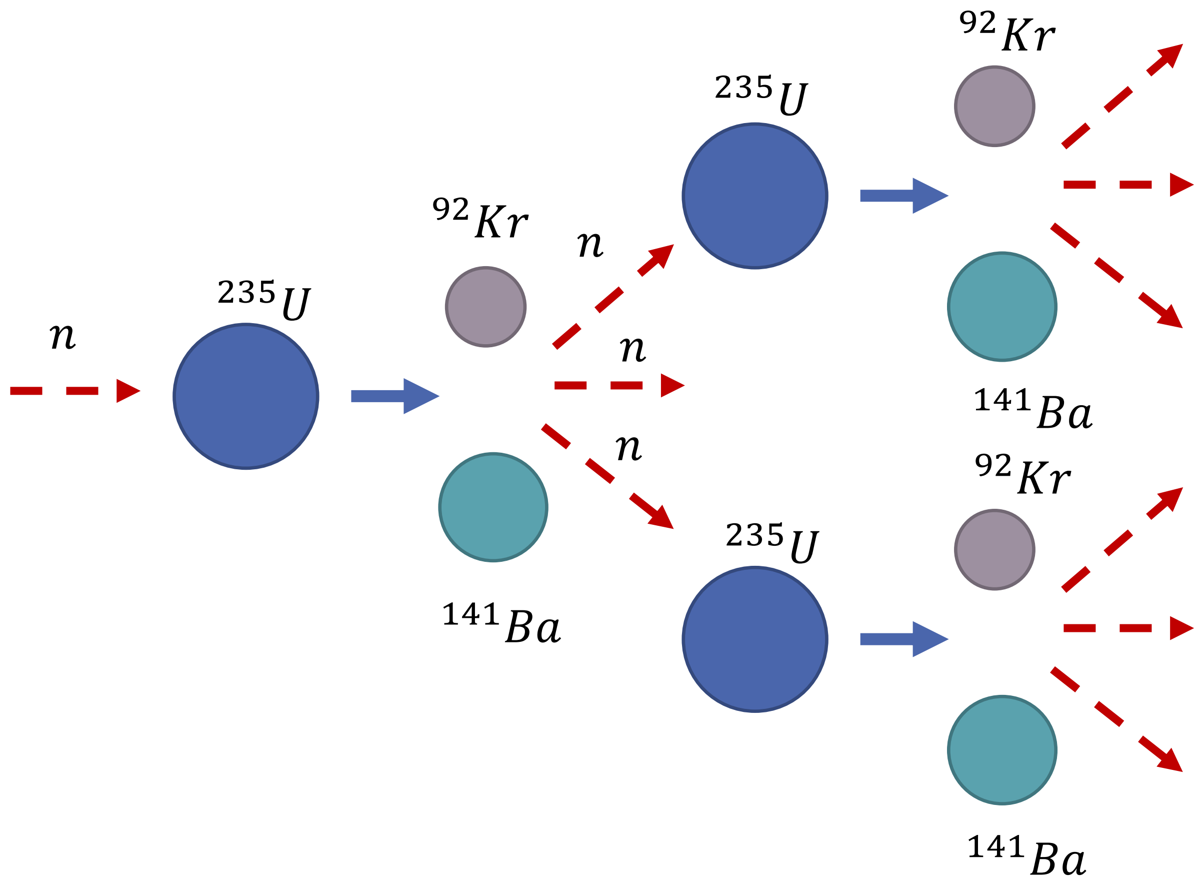
Heavy nuclei can absorb a neutron and then split into two smaller nuclei while releasing energy and more neutrons

For example, if ${}^{235}U$ nucleus absorbs a neutron it splits in ${}^{92}Kr$ and ${}^{141}Ba$ while releasing 202.5 MeV of energy

Under the right environment, this can set off a chain reaction

Controlled chain reaction gives a nuclear reactor, uncontrolled gives a nuclear bomb

$$n + {}^{235}U \rightarrow {}^{92}Kr + {}^{141}Ba + 3\,n + 202.5\,MeV$$



Nuclear Fusion

Likewise, light elements can fuse together to form heavier elements while giving off energy

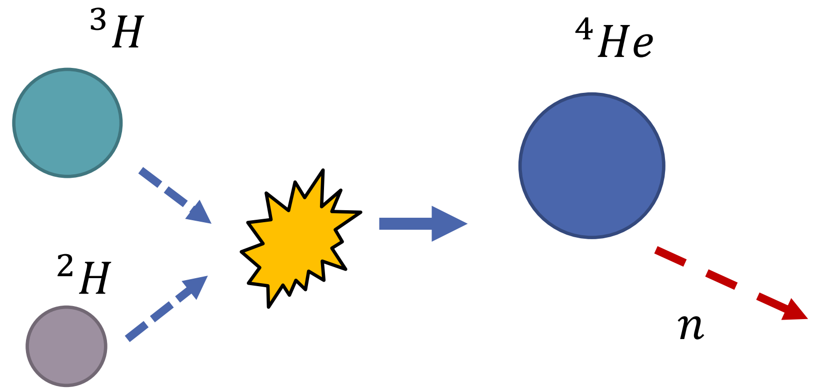
For example, if we take a deuterium (^{2}H) and tritium (^{3}H) nuclei and slam them together we get a helium nuclei (^{4}He), a neutron, and 17.6 MeV of energy

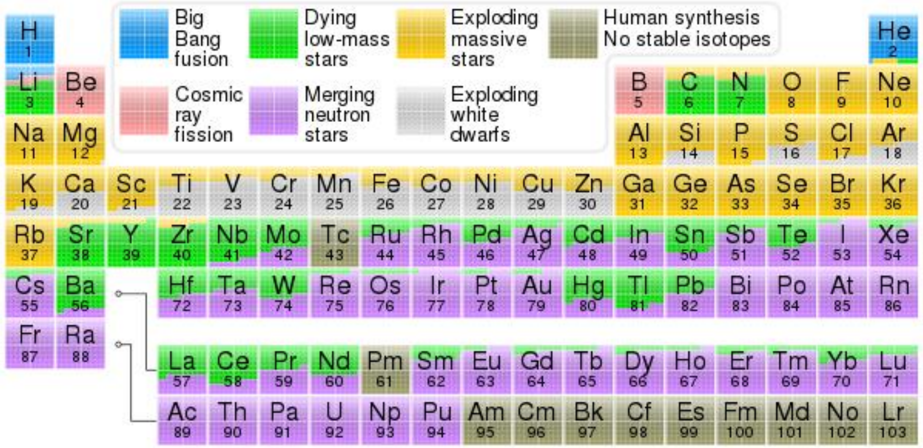
Fusion is what powers stars; gravity pulls the element together until they get hot enough to fuse and release energy which causes pressure that keeps the star from collapsing

All elements heavier than helium were created by fusion

Researchers are trying to make fusion reactors which would allow nuclear energy without radioactive waste

$$^{2}H + \ ^{3}H \rightarrow \ ^{4}He + n + 17.6\ MeV$$





Homework Questions

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