Phyx 320
Modern Physics

April 9, 2021

Reading: 41.1-41.4

Homework #10 Due Next Tuesday

Final Paper

For the final in this class, you will write a paper exploring a natural phenomenon or technological application related to the material we covered this semester. This can either be about a subject we explicitly covered in the course or a closely related topic. Treat the paper as a mock research paper that should thoroughly explore the subject in a concise and clear manner. Give enough background so that anyone from our class could understand it without much outside reading.

You are free to use any outside material that you like but you must cite all your sources. If you include direct quotes from a source, you must call it out explicitly and cite it.

Any text editor format is acceptable, but a pdf of the final paper is preferred. For those who are planning to pursue an academic or research career, I would highly recommend getting used to using LaTeX.

Example topics: the twin paradox, the physics of a nuclear reactor, exoplanet atmosphere spectroscopy, Heisenberg's uncertainty principle, effects of travel at nearly the speed of light, history of atomic models, radioisotope dating

Suggested length: 2-4 pages. Total grade: 25 pts.

Final Paper

For full credit, the paper must meet the following conditions:

1. Contains a derivation or calculation related to the subject (5 pts.) Every variable must be defined in the text of the paper. You must show every step of the calculation or include the code used for the calculation in the code section (see 5).

2. Cites material used and contains a bibliography (5 pts.) Any bibliography style is acceptable. You must cite every source that you used including our textbook and call out any direct quotes.

3. Include at least one plot related to the subject (5 pts.) Plot should be readable and clearly express the point of the plot. The axes must be labeled with correct units.

4. Concisely but thoroughly explores the topic of interest (5 pts.) Give a concise introduction to the subject. Explore some aspect in detail. The paper does not need to be all-encompassing, but it should stand on its own. Finish the paper with a concise conclusion.

5. Includes all of the code used to make plots or do calculations (5 pts.) Copy and paste your raw code at the end of the paper. Any programming language is acceptable. Must be organized and contain comments describing each step.

Hydrogen Atom

Valid solutions to the hydrogen atom following three conditions:

1. Energy is quantized and depends on the principal quantum number, n :

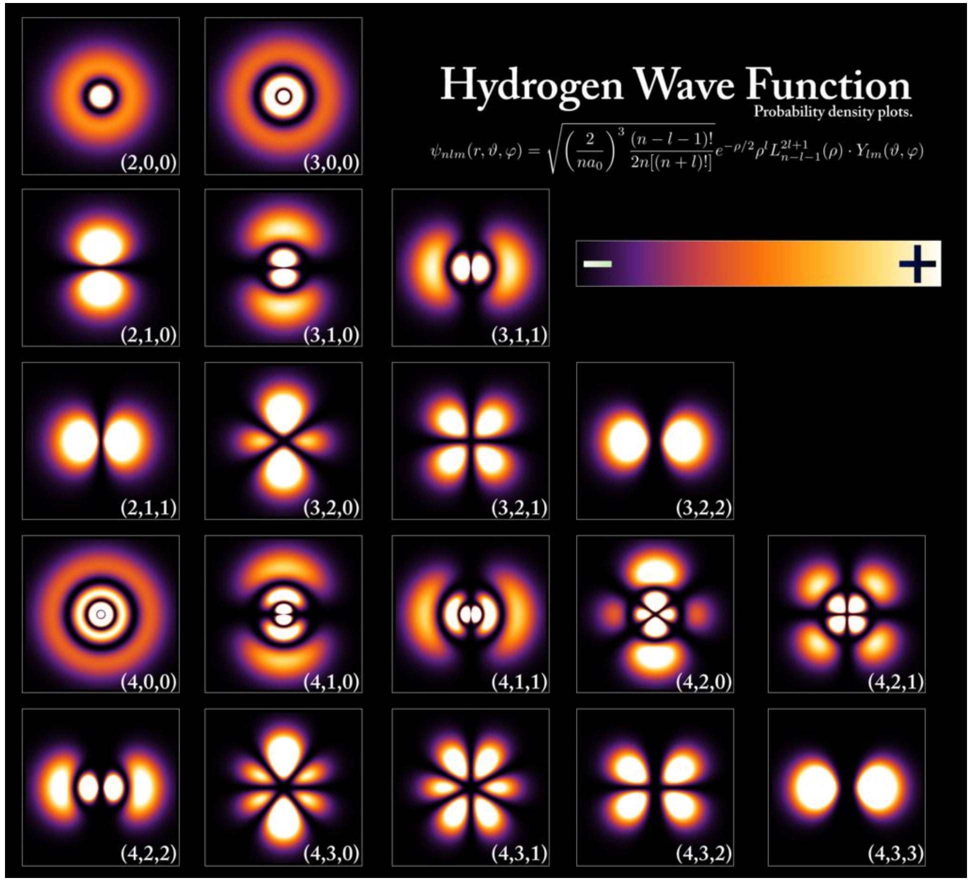
$$E_n = -\frac{1}{n^2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{2a_b} \qquad n = 1, 2, 3, \ldots$$

2. Orbit angular moment is determined with the orbital quantum number, l:

$$L = \sqrt{l(l+1)}\hbar \qquad l = 1, 2, \ldots, n-1$$

3. z-component of the angular moment controlled by magnetic quantum number, m:

$$L_z = m\hbar \qquad m = -l, -l+1, \ldots, l-1, l$$



Magnetic Moments

When we have an electric charge orbiting in circular motion, we get a loop of current

A loop of current induces a magnetic moment, $\vec{\mu}$

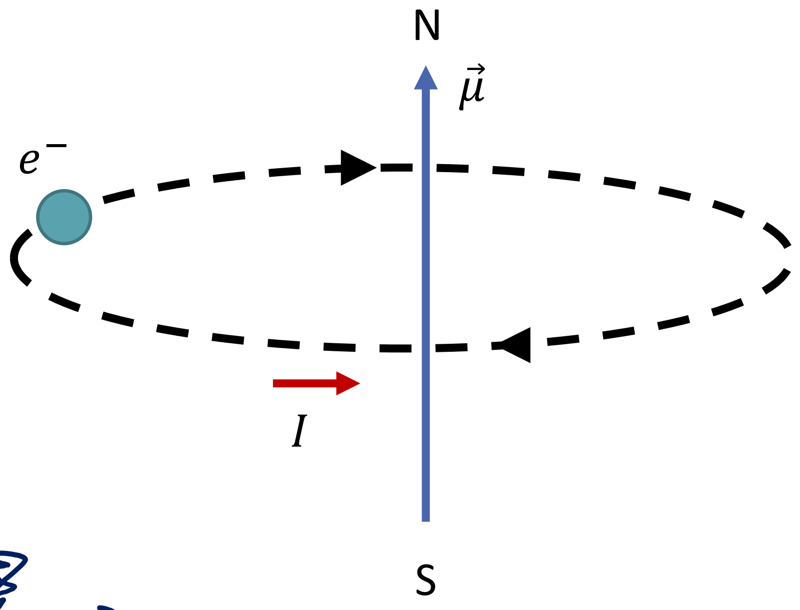
Magnetic moments interaction with external magnetic fields to give a potential energy:

$$U = -\vec{\mu} \cdot \vec{B}$$

Non-uniform magnetic fields apply both torques and forces on magnetic moments

For orbital angular moment:

$$\vec{\mu} \propto \vec{L}$$



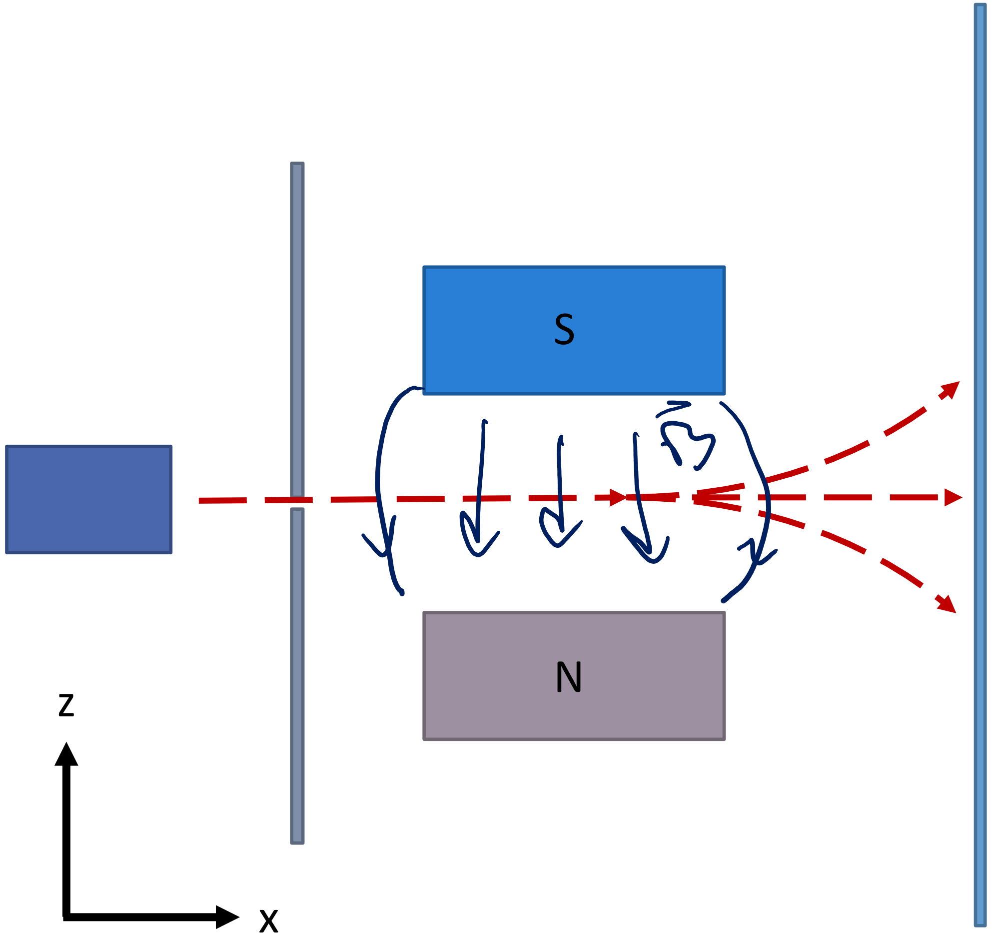
Stern-Gerlach Experiment

Stern-Gerlach experiment measures magnetic moments by sending a beam of particles through a non-uniform magnetic field

Measures the z-component of the magnetic moment

For atoms, $\vec{\mu} \propto \vec{L}$, so this measures the z-component of the orbital angular moment

If $\vec{\mu}$ is pointed up ($\mu_z > 0$) the atom feels an upward force, if down ($\mu_z < 0$) feels a downward force



Stern-Gerlach Experiment

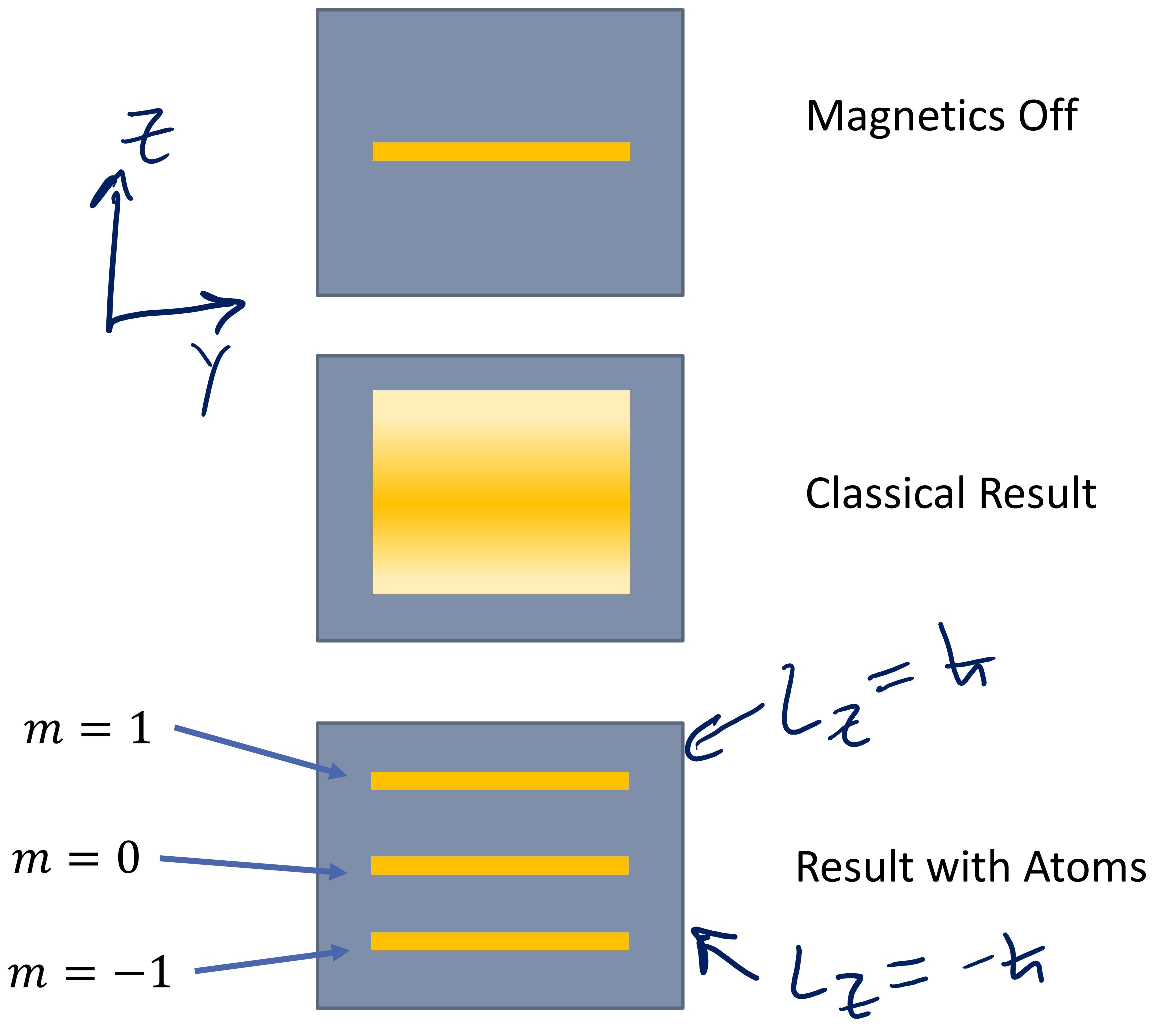
With magnets off the beam goes straight through and shows up on the screen as a single line

Classically we'd expect to find a smooth gradient since L_z can have any value

However, we know from the previous lecture that $L_z = m\,\hbar$ with $m = -l, \dots, l$

What we find is the for $l = 1$ atoms we get three different strips corresponding to:

$$m = -1, 0, 1$$



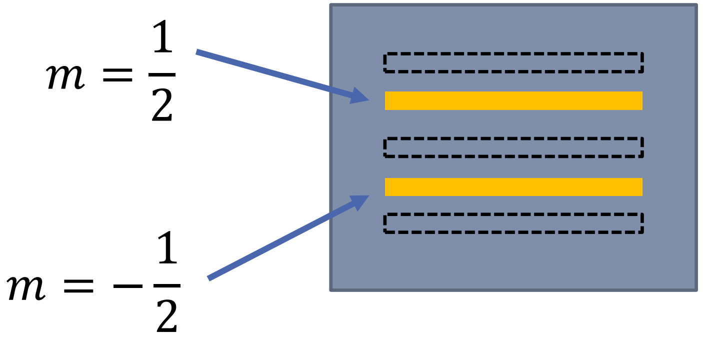
Stern-Gerlach Experiment

Now if we send through only electrons, we get two stripes instead three

None of the particles hit in the center

This implies that the electron has an inherent magnetic moment and thus inherent angular momentum

Exactly half of the orbital angular moment of the hydrogen atom



Electron Spin

We call this inherent angular momentum 'spin'

Electron is not actually spinning but the classically analogy is a top spinning and moving through space

Electrons always have angular momentum

Controlled by a new set of quantum numbers: m_s, s

All fundamental particles have spin

Particles now have three properties: mass, charge, and spin

$$S_z = m_s \hbar$$

$$S = \sqrt{s(s+1)}\hbar$$

$|\vec{S}|$

$= \sqrt{(3/4)}\,\hbar$

For electrons:

$$m_s = -\frac{1}{2}, \frac{1}{2}$$

$$s = \frac{1}{2}$$

For other particles:

$$m_s = -s, -s+1, \ldots s-1, s$$

$$s = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

Multielectron Atoms

Let's return to elements with more than one proton ($Z > 1$)

The potential that the electron feels can be approximated as:

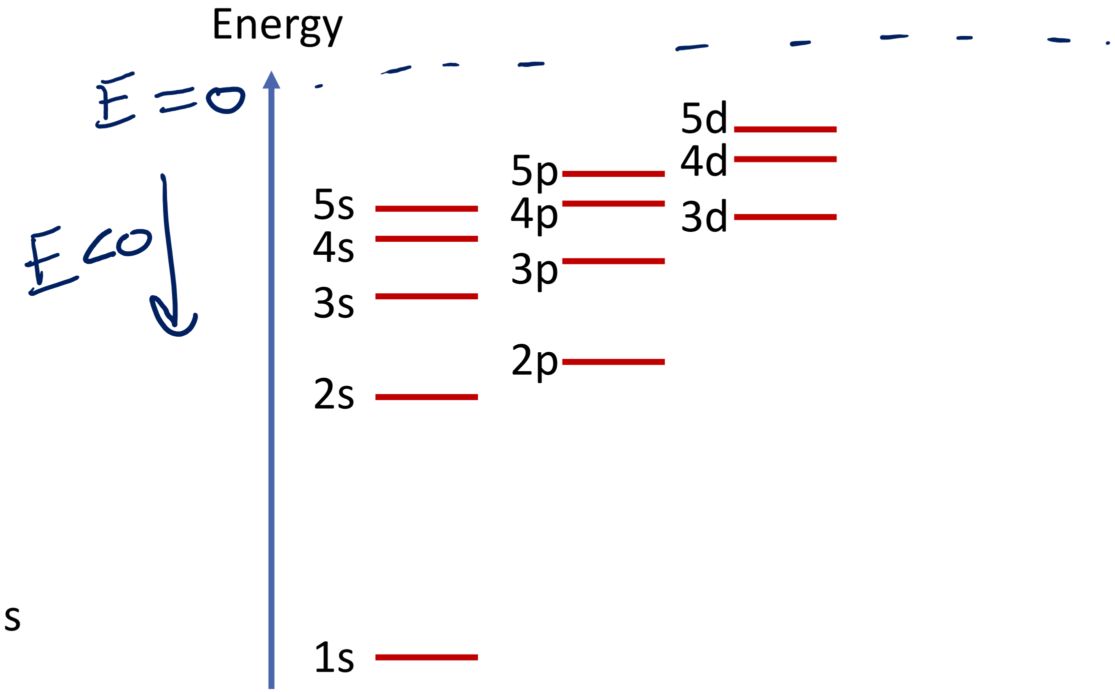
$$U(r) = -\frac{Ze^2}{4\pi\epsilon_0 r} + U_{elec}(r)$$

Nucleus

Other electrons

Due to the other electrons, the energy depend on both n and l

Higher-l means higher energy



Pauli Exclusion Principle

One consequence of electron spin is that no two electrons can have the exact same quantum numbers

Pauli exclusion principle: if an electron is in a given state, it excludes all other electron from occupying that state

Since electrons only have two spin states, we can call on up ($\uparrow = \frac{1}{2}\hbar$) and down ($\downarrow = -\frac{1}{2}\hbar$)

s-subshells ($l = 0$) have two states:

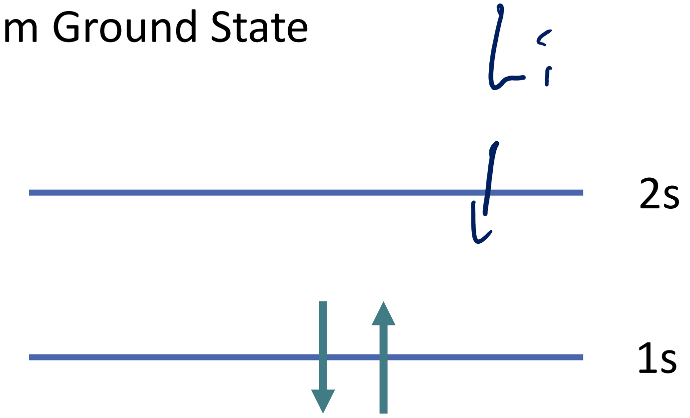
$$(n, 0, 0, \uparrow), (n, 0, 0, \downarrow)$$

p-subshells ($l = 1$) have six states:

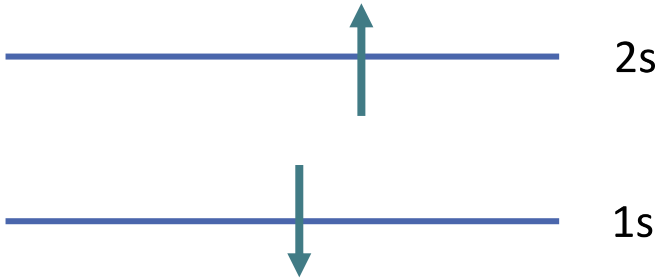
$$(n, 1, -1, \uparrow), (n, 1, -1, \downarrow), (n, 1, 0, \uparrow),$$

$$(n, 1, 0, \downarrow), (n, 1, 1, \uparrow), (n, 1, 1, \downarrow)$$

Helium Ground State



Helium Excited State



~~Homework Questions~~ muon g-2

$m_e = 0.5$ MeV

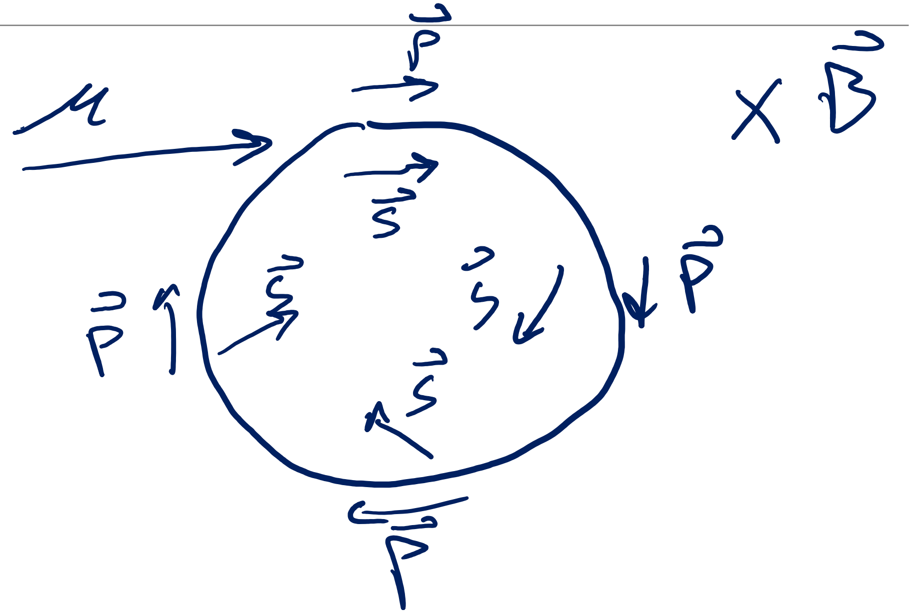
$m_\mu = 105.7$ MeV

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

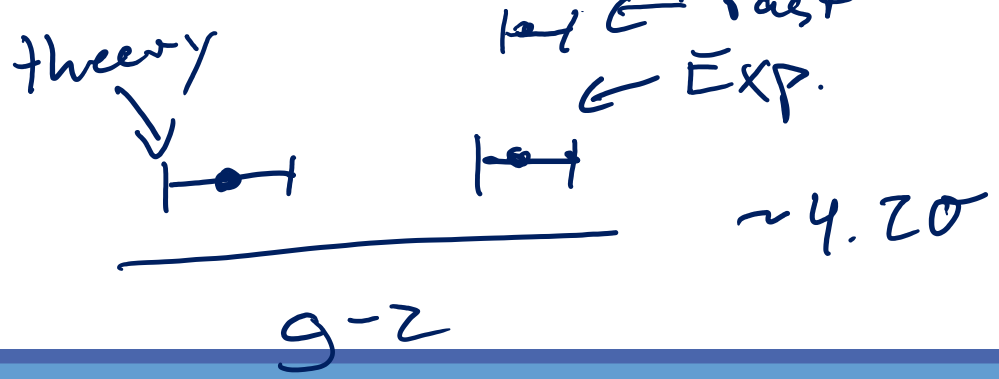
mag. moment

QM ⇒ g = 2

SM ⇒ g ≈ 2



g = 2: p ⇉ s parallel



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