Phyx 320
Modern Physics

April 2, 2021

Reading: 41.1-41.4

Homework #10 Due Next Tuesday

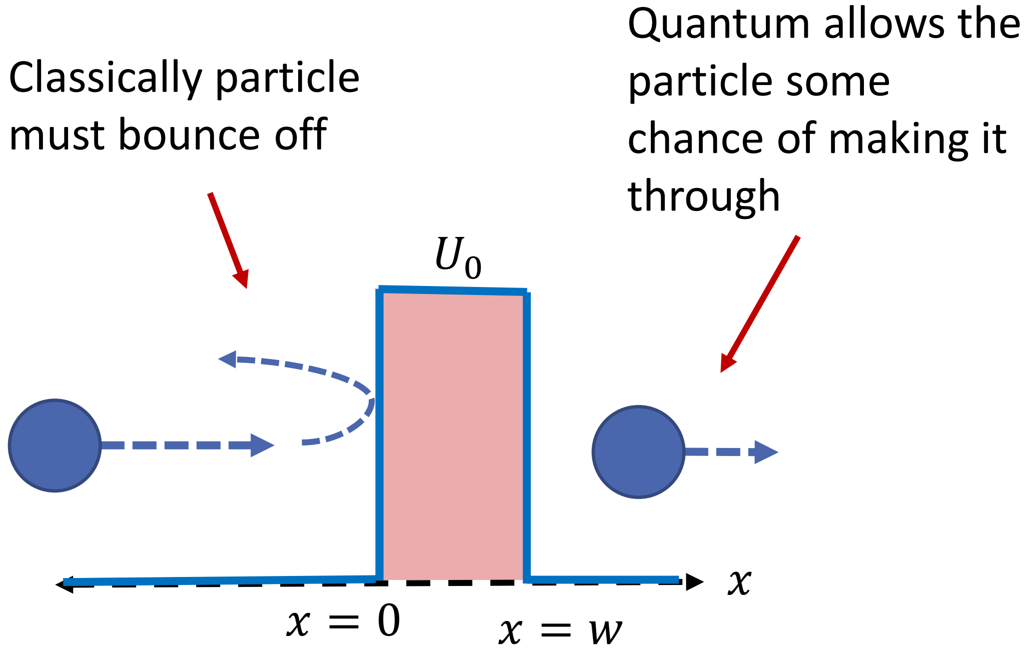
Quantum Tunneling

Particles can "tunnel" through potential barrier that would classically be forbidden

$$P_{tunnel} = e^{-\frac{2w}{\eta}}$$

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

Chance of tunneling depends on the height and width of the potential and the energy of the incoming particle



Hydrogen Atom

Now that we have a grasp on the Schrödinger Equation, let's return to the hydrogen atom

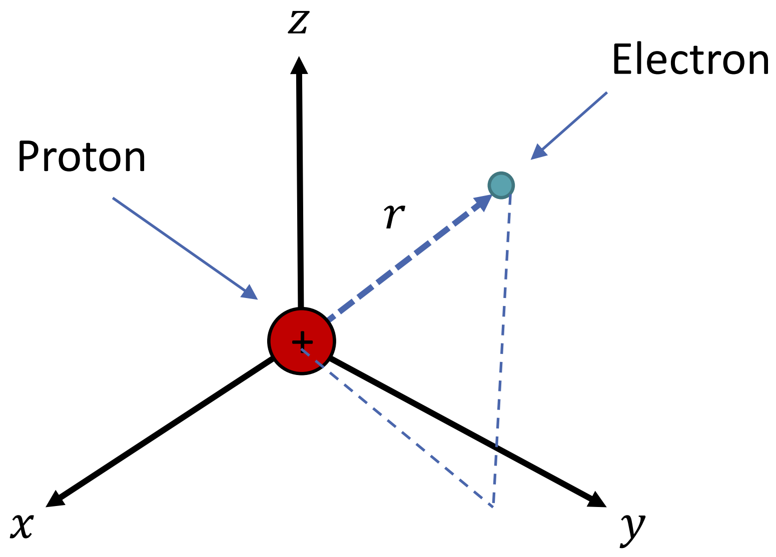
Hydrogen atom is a single proton with a single electron orbiting it

The proton is roughly 2,000 times more massive than the electron so we will treat it as stationary

We'll center our coordinate system on the proton and working in spherical coordinates

The potential that the electron experiences follows:

$$U(r) = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$



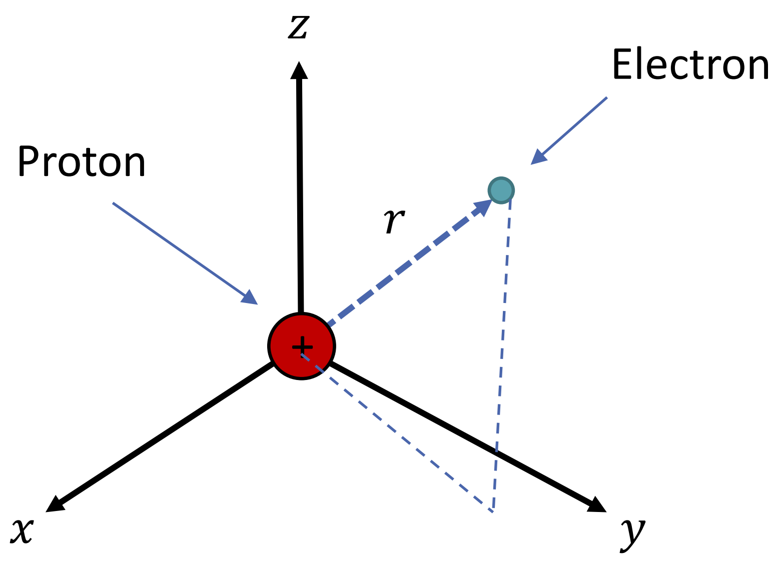
Hydrogen Atom

To accurately analyze the hydrogen atom, we must work in three-dimensions

The full derivation will have to wait for a higher-level quantum mechanics course, but we can still analyze the results

We found that in one-dimension wavefunctions were labeled with one quantum number, n

In three dimensions we need three: n, l, m



Hydrogen Atom

Valid solutions to the hydrogen atom following three conditions:

1. Energy is quantized and depends on the principal quantum number, n :

$$E_n = -\frac{1}{n^2}\frac{1}{4\pi\epsilon_0}\frac{e^2}{2a_b} \qquad n = 1, 2, 3, \ldots$$

2. Orbit angular moment is determined with the orbital quantum number, l:

$$L = \sqrt{l(l+1)}\hbar \qquad l = 1, 2, \ldots, n-1$$

3. z-component of the angular moment controlled by magnetic quantum number, m:

$$L_z = m\hbar \qquad m = -l, -l+1, \ldots, l-1, l$$

Label	l
s	0
p	1
d	2
f	3

State	(n, l, m)
1s	(1,0,0)
2s	(2,0,0)
2p	(2,1,m)
3s	(3,0,0)
3p	(3,1,m)
3d	(3,2,m)

Angular Momentum

Classically angular momentum can have any value and point in any direction

Quantum mechanics only allows specific values of $L = |\vec{L}|$ and L_z

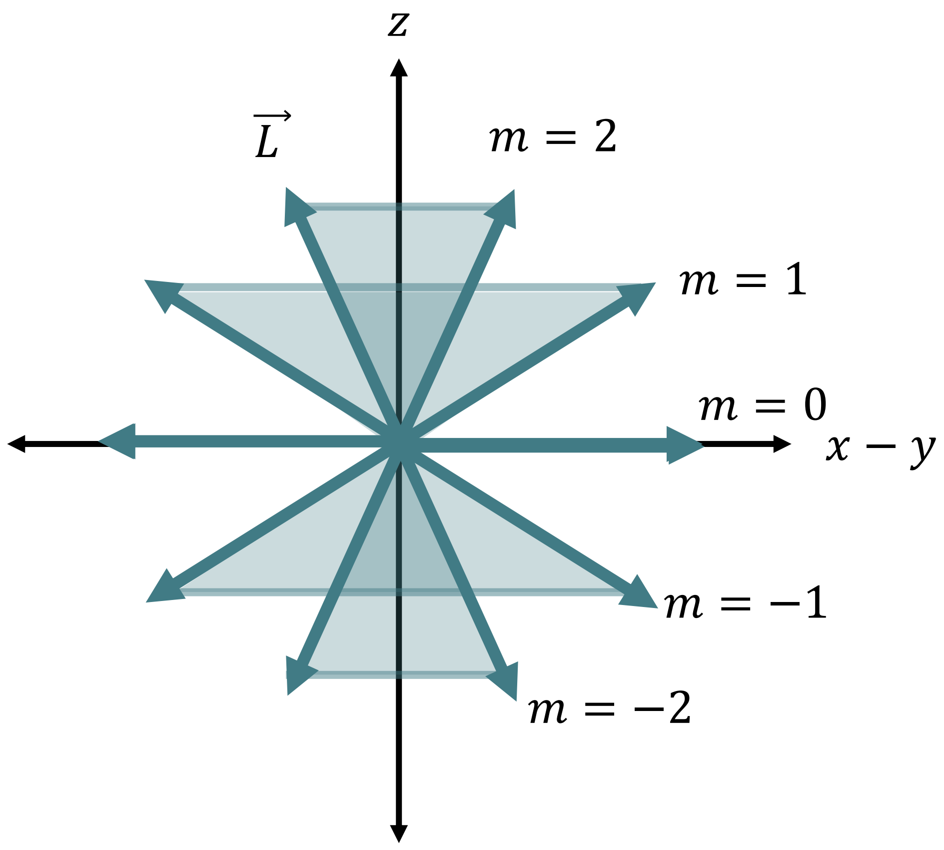
Looking at the $n = 3$ states, $l = 0,1,2$:

$$L = 0, \sqrt{2}\hbar, \sqrt{6}\hbar$$

Focusing on the $l = 2$ state, $m = -2, -1, 0, 1, 2$

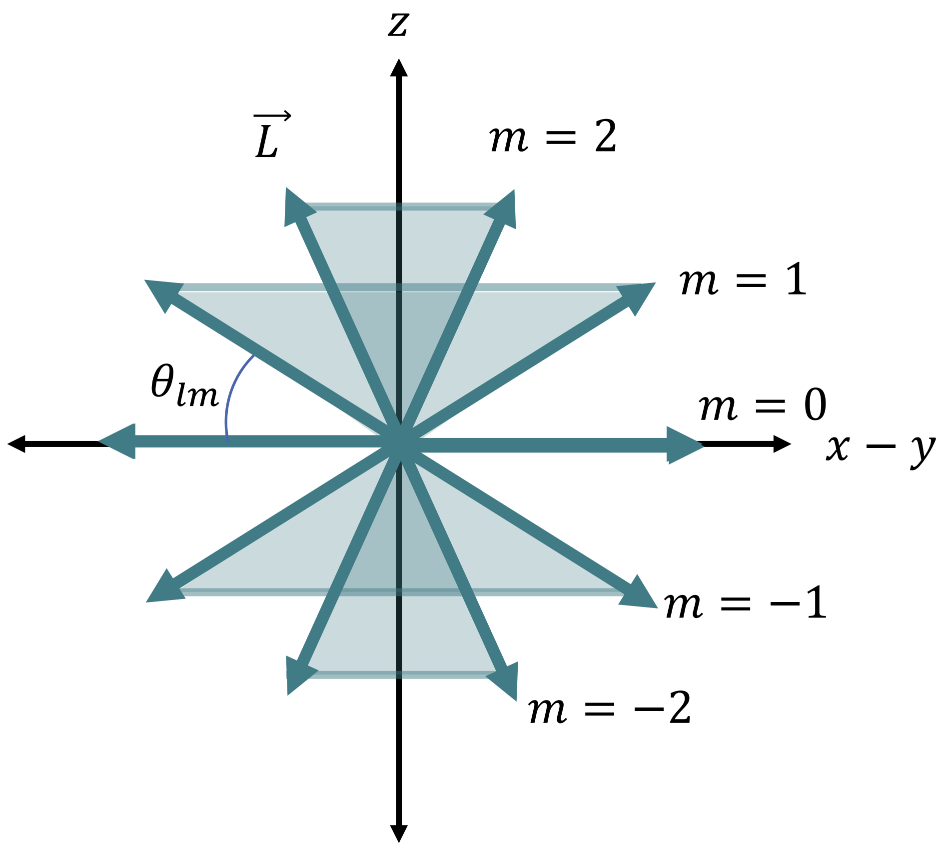
$$L_z = -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar$$

Since $\mathrm{L} = \sqrt{6}\hbar = 2.45\hbar$ and the maximum of $L_z = 2\hbar$, the vector $\vec{L}$ can not point in purely the z-direction



Angular Momentum

The orbital plane is restricted to have only a few discrete values:



Wave Function

Since we're working in three-dimensions, we must reevaluate what our probability densities mean:

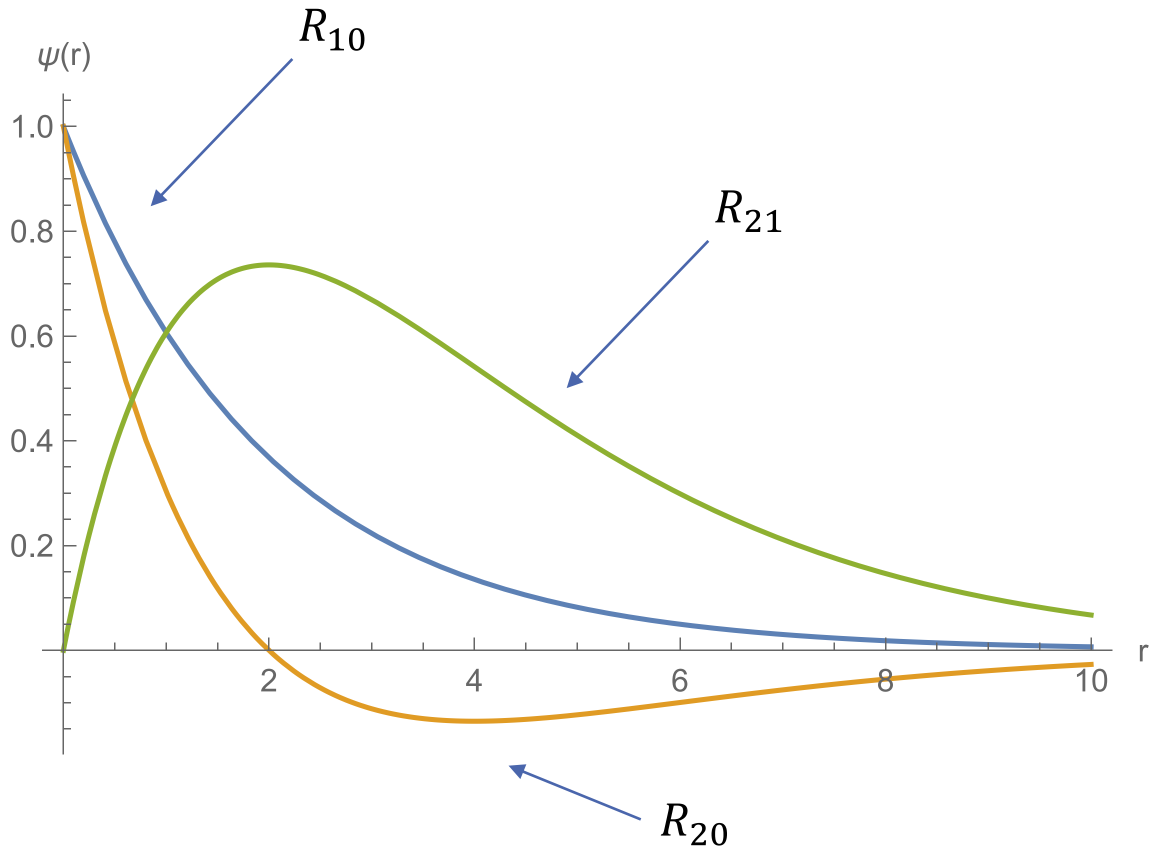
Wave Function

Radial wave functions:

$$R_{10}(r) = \frac{1}{\sqrt{\pi a_B^3}} e^{-\frac{r}{a_B}}$$

$$R_{20}(r) = \frac{1}{\sqrt{8\pi a_B^3}} \left(1 - \frac{r}{2a_B}\right) e^{-\frac{r}{a_B}}$$

$$R_{21}(r) = \frac{1}{\sqrt{24\pi a_B^3}} \left(\frac{r}{2a_B}\right) e^{-\frac{r}{a_B}}$$



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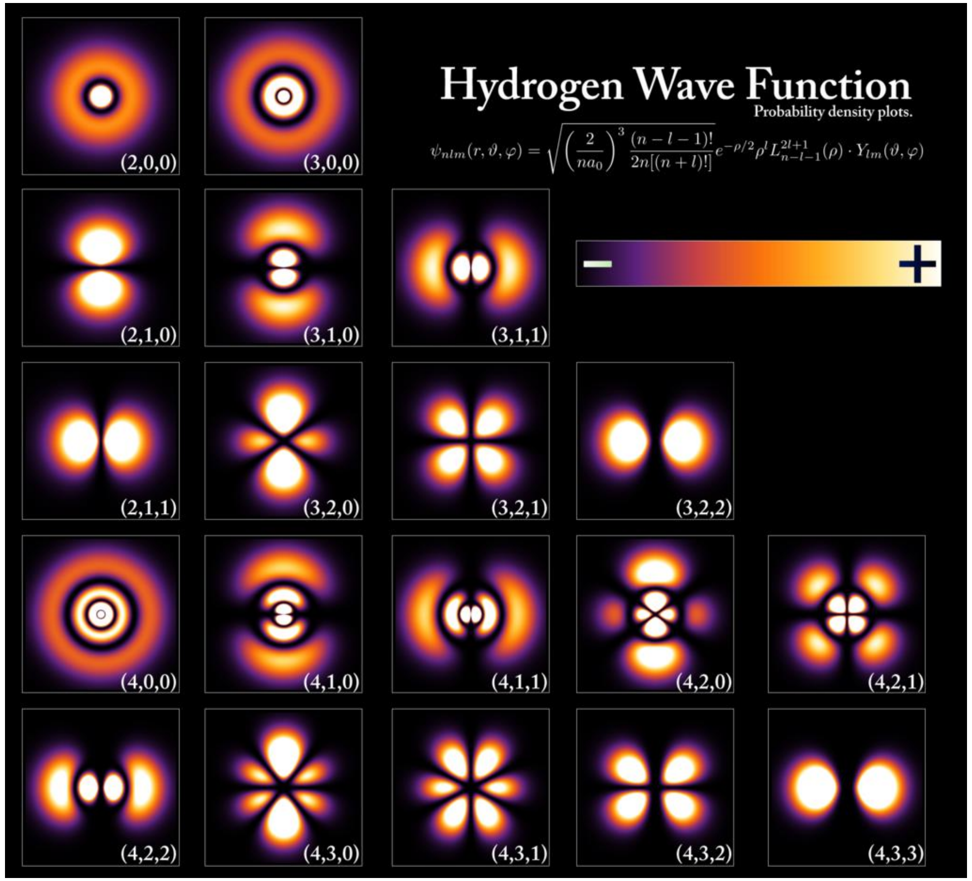

Homework Questions

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