Phyx 320 Modern Physics

April 2, 2021

Reading: 40.8-40.10

No Homework or Reading Reflection Due Next Week

Harmonic Oscillator

Higher states:

$$\psi_1(x) = A_1 e^{-\frac{x^2}{2b^2}}$$

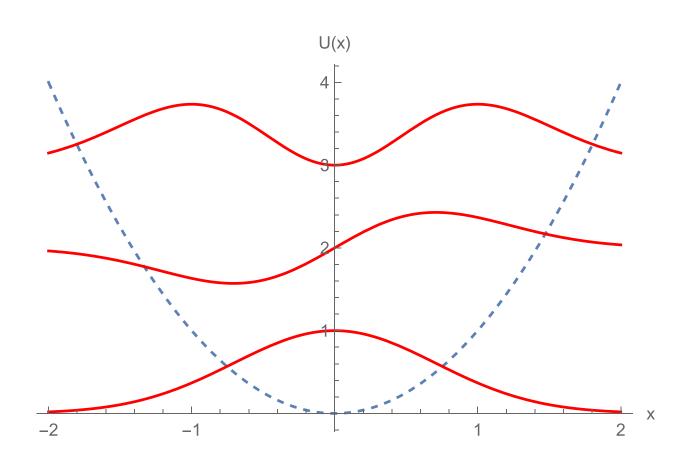
$$\psi_2(x) = A_2 \frac{x}{b} e^{-\frac{x^2}{2b^2}}$$

$$\psi_3(x) = A_3 \left(1 - \frac{2x^2}{b^2} \right) e^{-\frac{x^2}{2b^2}}$$

$$b = \sqrt{\frac{\hbar}{m\omega}}$$

Energies follow:

$$E_n = \left(n - \frac{1}{2}\right)\hbar\omega$$



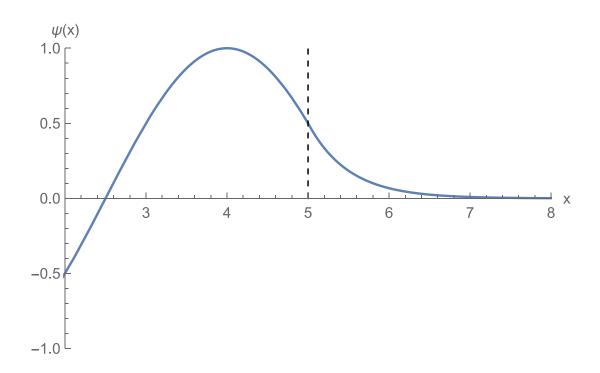
If we think back to the finite potential well, there was some probability that the particle could be found inside the classically forbidden region

In classically forbidden region, wavefunction decays exponentially:

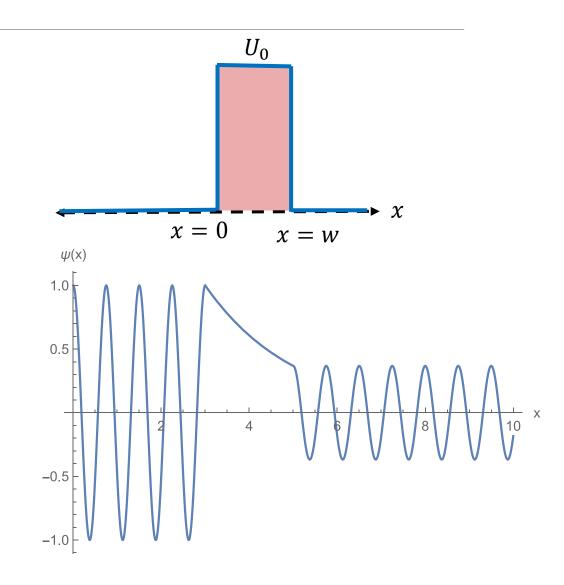
$$\psi(x) = \psi_{edge} \, e^{-\frac{x-L}{\eta}}$$

Wavefunction decays with a characteristic length scale, penetration depth:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$



What happens if we make the potential a finite width?



Ensuring continuity at boundaries:

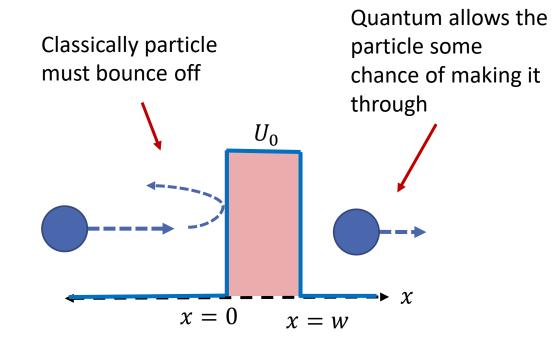
If we send in a particle from the left, what probability would it end up on the right of the barrier?

Particles can "tunnel" through potential barrier that would classically be forbidden

$$P_{tunnel} = e^{-\frac{2w}{\eta}}$$

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

Chance of tunneling depends on the height and width of the potential and the energy of the incoming particle



Any "trapped" quantum particle has a probability of escaping

Has wide ranging implications

- Scanning tunneling microscope: able to image surfaces at the atomic level
- Spontaneous DNA mutation: protons can tunnel to new location making DNA mutate
- Radioactive decay: protons and neutrons can tunnel out of nucleus

