Phyx 320
Modern Physics

HW Due on Tuesdays Again

April 2, 2021

Reading: 40.8-40.10

No Homework or Reading Reflection Due Next Week

Harmonic Oscillator

Higher states:

$$\psi_1(x) = A_1 e^{-\frac{x^2}{2b^2}}$$

$$\psi_2(x) = A_2 \frac{x}{b} e^{-\frac{x^2}{2b^2}}$$

$$\psi_3(x) = A_3 \left(1 - \frac{2x^2}{b^2}\right) e^{-\frac{x^2}{2b^2}}$$

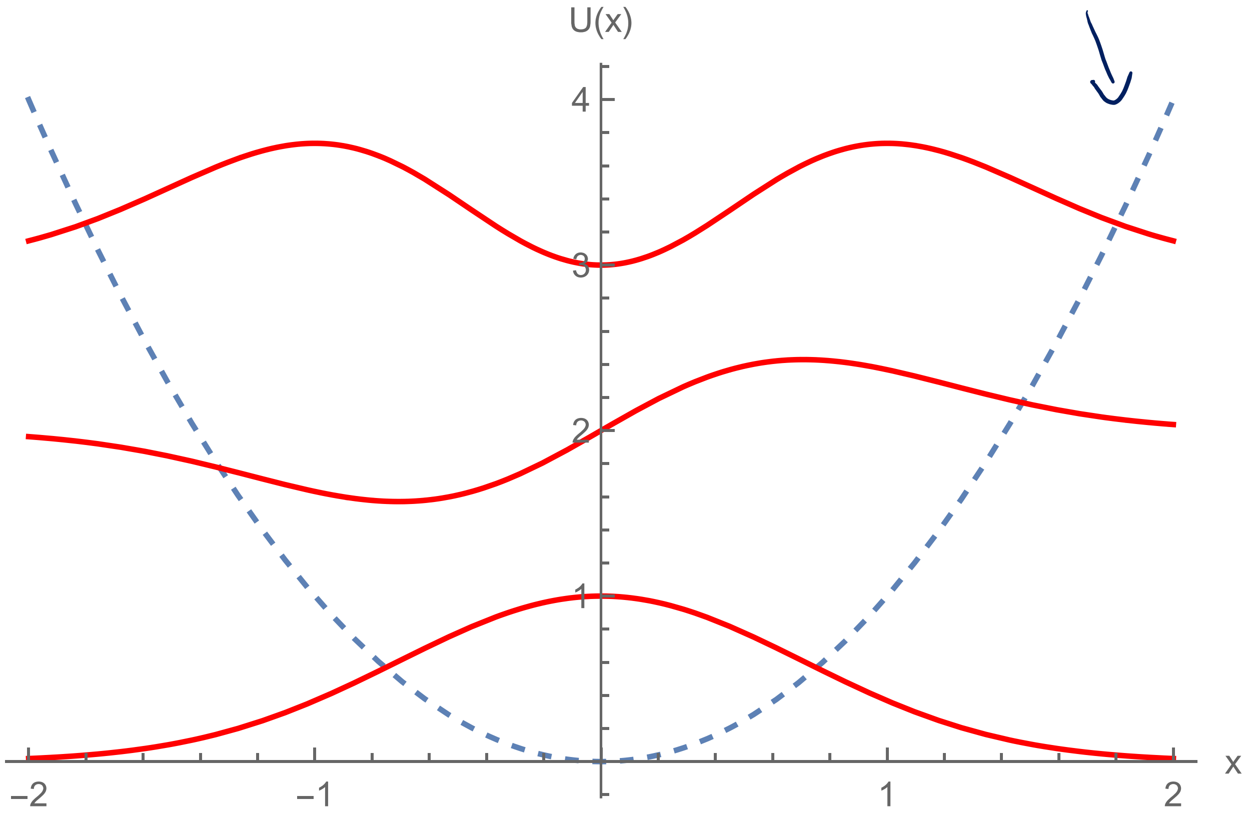
$$b = \sqrt{\frac{\hbar}{m\omega}}$$

$$\omega = \sqrt{k/m}$$

Energies follow:

$$E_n = \left(n - \frac{1}{2}\right)\hbar\omega$$

$$U(x) = \frac{1}{2} k x^2$$



Quantum Tunneling

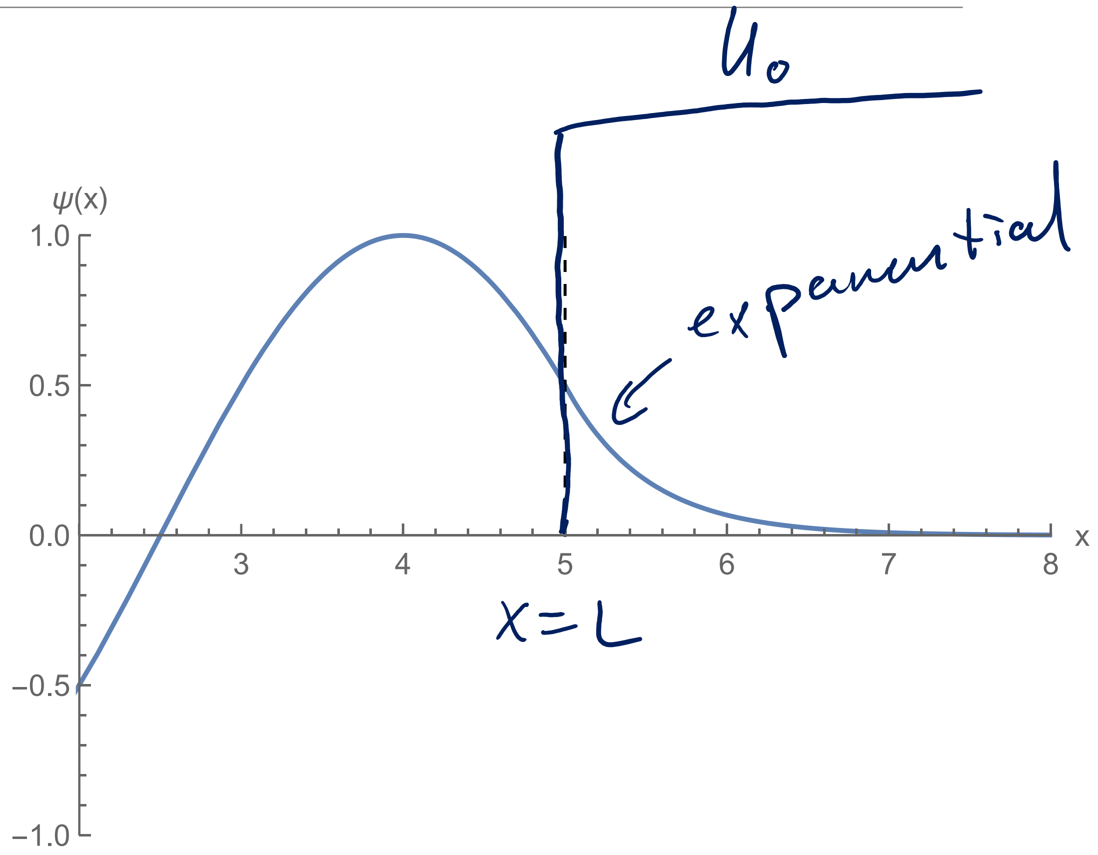
If we think back to the finite potential well, there was some probability that the particle could be found inside the classically forbidden region

In classically forbidden region, wavefunction decays exponentially:

$$\psi(x) = \psi_{edge}\, e^{-\frac{x-L}{\eta}}$$

Wavefunction decays with a characteristic length scale, penetration depth:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$



Quantum Tunneling

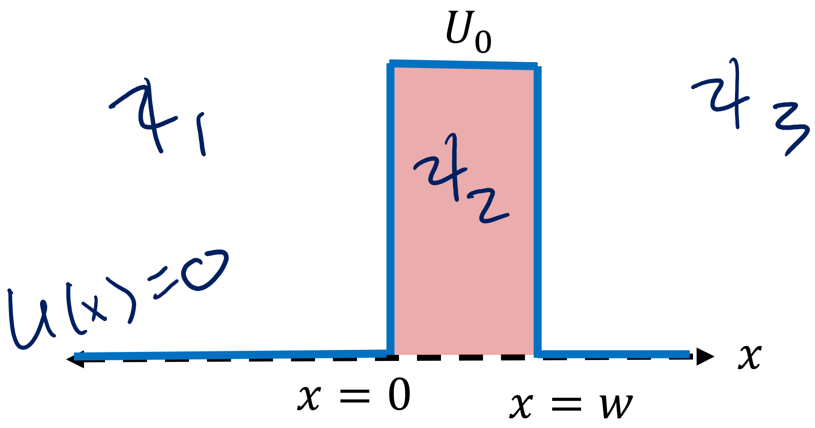
What happens if we make the potential a finite width?

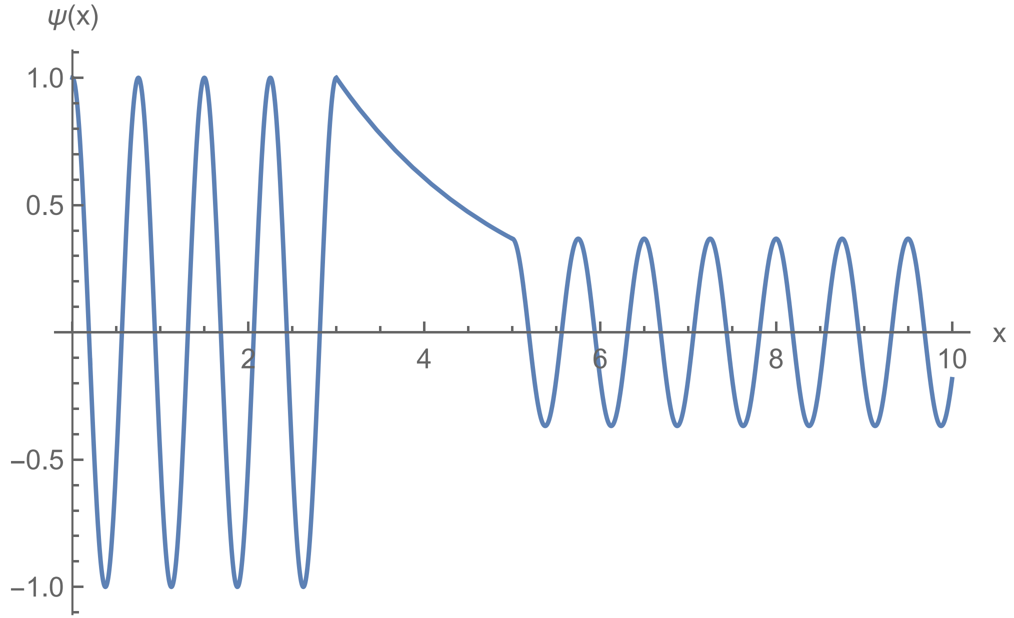
$$\psi_1(x<0) = A_L \cos(2\pi x/\lambda)$$

$$\psi_3(x>w) = A_R \cos\left(2\pi \frac{(x-w)}{\lambda}\right)$$

$$\psi_2(0 \le x \le w) = \psi_{edge}\, e^{-x/\eta}$$

$$\eta = \frac{\hbar}{\sqrt{2m(U_0-E)}}$$





Quantum Tunneling

Ensuring continuity at boundaries:

$x = 0$:

$$\psi_1(x=0) = \psi_2(x=0)$$

$$\psi_1(x=0) = A_L \overbrace{\cos(0)}^{1}$$

$$= A_L$$

$$\psi_2(x=0) = \psi_{edge} \overbrace{e^{-0/\eta}}^{1}$$

$$\boxed{\psi_{edge} = A_L}$$

$x = w$:

$$\psi_2(x=w) = \psi_3(x=w)$$

$$\psi_2(x=w) = A_L e^{-w/\eta}$$

$$\psi_3(x=w) = A_R \overbrace{\cos\left(2\pi \frac{\cancel{w-w}^{\,0}}{L}\right)}^{1}$$

$$\boxed{A_R = A_L e^{-w/\eta}}$$

Quantum Tunneling

If we send in a particle from the left, what probability would it end up on the right of the barrier?

$$P_L \propto |A_L|^2$$

$$P_R \propto |A_R|^2$$

$$P_{tunnel} = \frac{P_R}{P_L} = \frac{A_R^2}{A_L^2}$$

$$= \frac{\left(A_L e^{-w/\eta}\right)^2}{A_L^2}$$

$$= \frac{A_L^2 e^{-2w/\eta}}{A_C^2}$$

$$= e^{-2w/\eta}$$

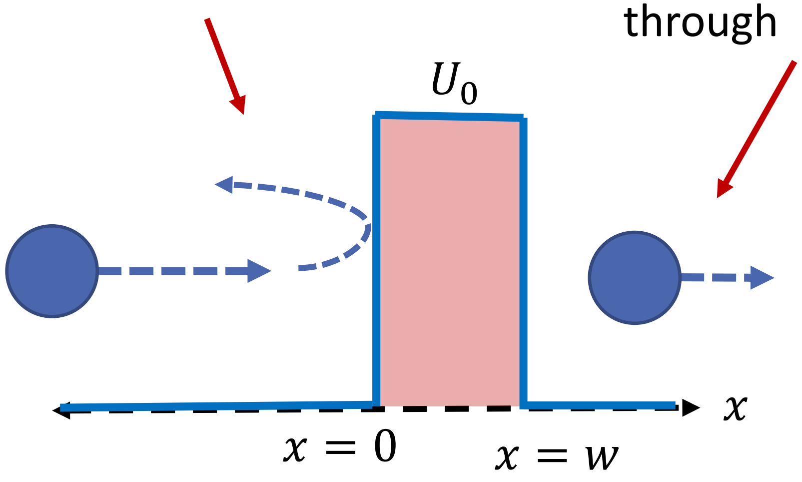
Quantum Tunneling

Particles can "tunnel" through potential barrier that would classically be forbidden

$$P_{tunnel} = e^{-\frac{2w}{\eta}}$$

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$

Chance of tunneling depends on the height and width of the potential and the energy of the incoming particle



$$E \downarrow \Rightarrow \eta \uparrow \Rightarrow \frac{2w}{\eta} \downarrow \Rightarrow P \uparrow$$

Quantum Tunneling

Any "trapped" quantum particle has a probability of escaping

Has wide ranging implications

- Scanning tunneling microscope: able to image surfaces at the atomic level
- Spontaneous DNA mutation: protons can tunnel to new location making DNA mutate
- Radioactive decay: protons and neutrons can tunnel out of nucleus

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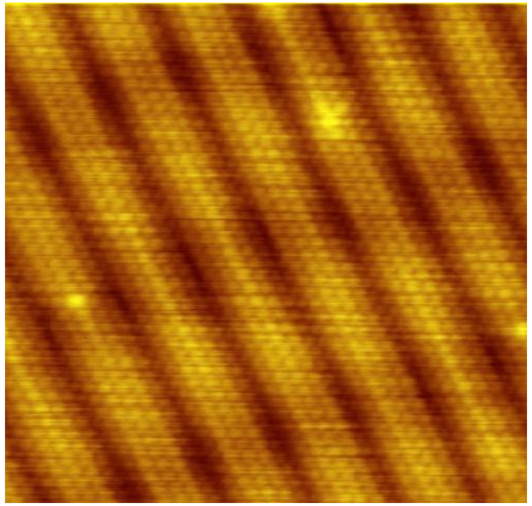

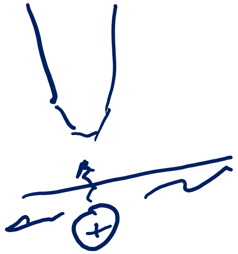

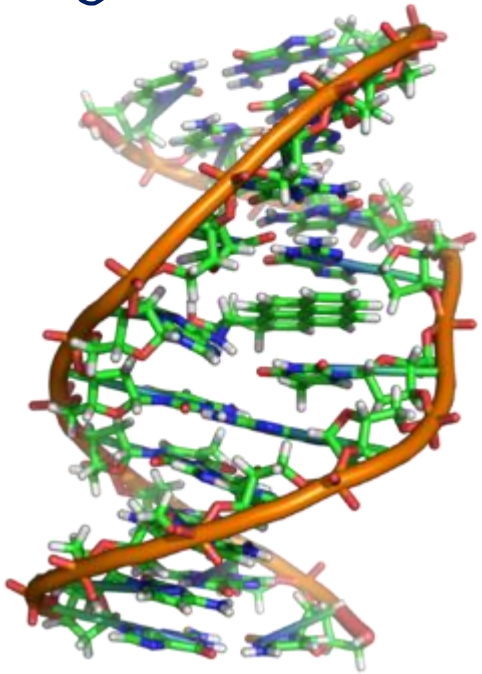

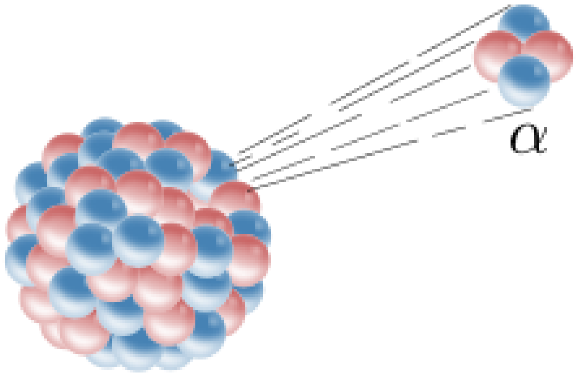

Homework Questions

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