Phyx 320
Modern Physics

March 29, 2021

Reading: 40.5-40.8

Homework #9 and Reading Reflection Thursday 11:59 pm

Finite Potential Wells

Two types of states:

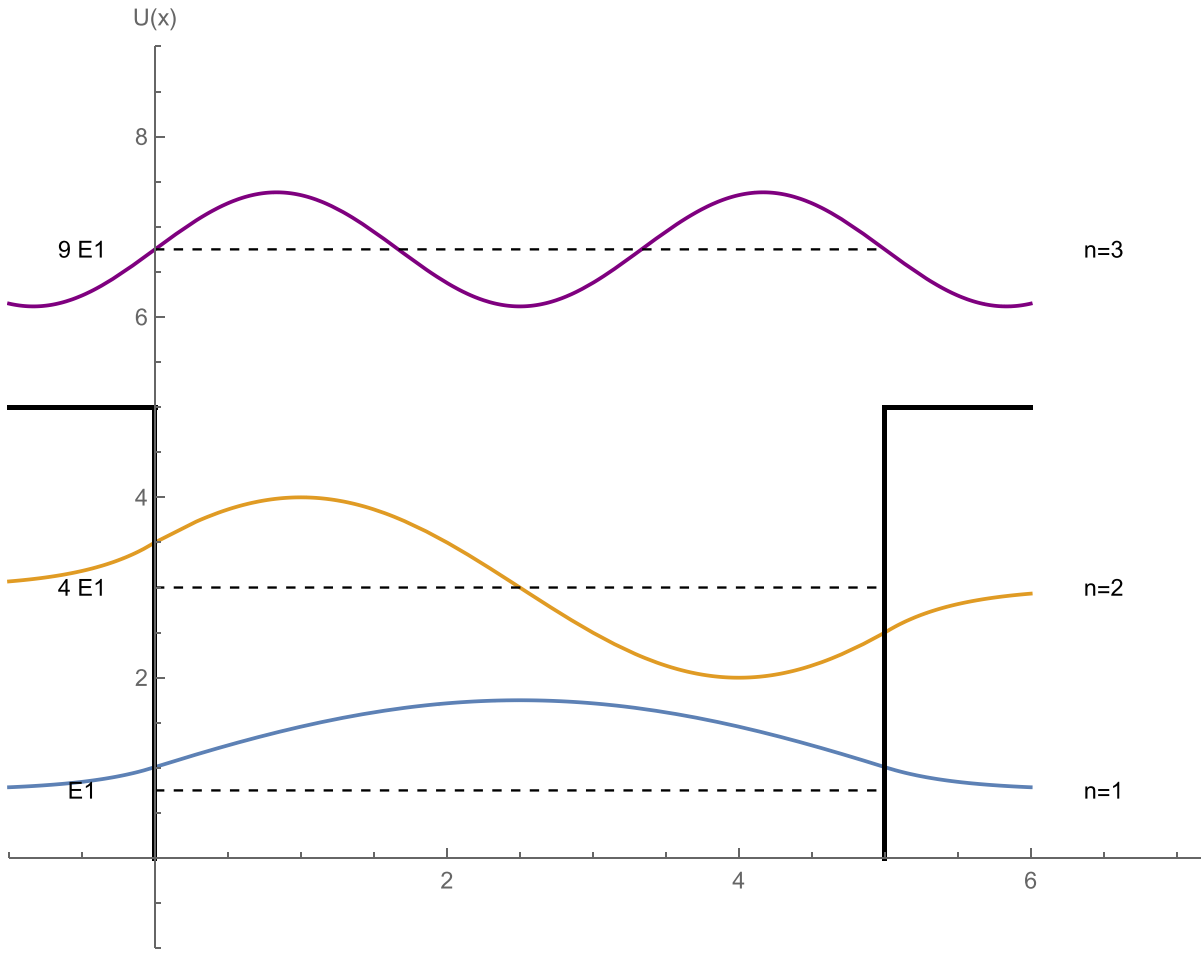
- Bound states ($E < U_0$)
- Free states ($E > U_0$)

In classically forbidden region, wavefunction decays exponentially:

$$\psi(x) = \psi_{edge}\, e^{-\frac{x-L}{\eta}}$$

Wavefunction decays with a characteristic length scale, penetration depth:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$



Harmonic Oscillator

Harmonic potential:

$$\mathrm{U}(x) = \frac{1}{2}\kappa\, x^2$$

Classical systems:

- Mass on a spring
- Pendulums
- Acoustic systems
- Circuits

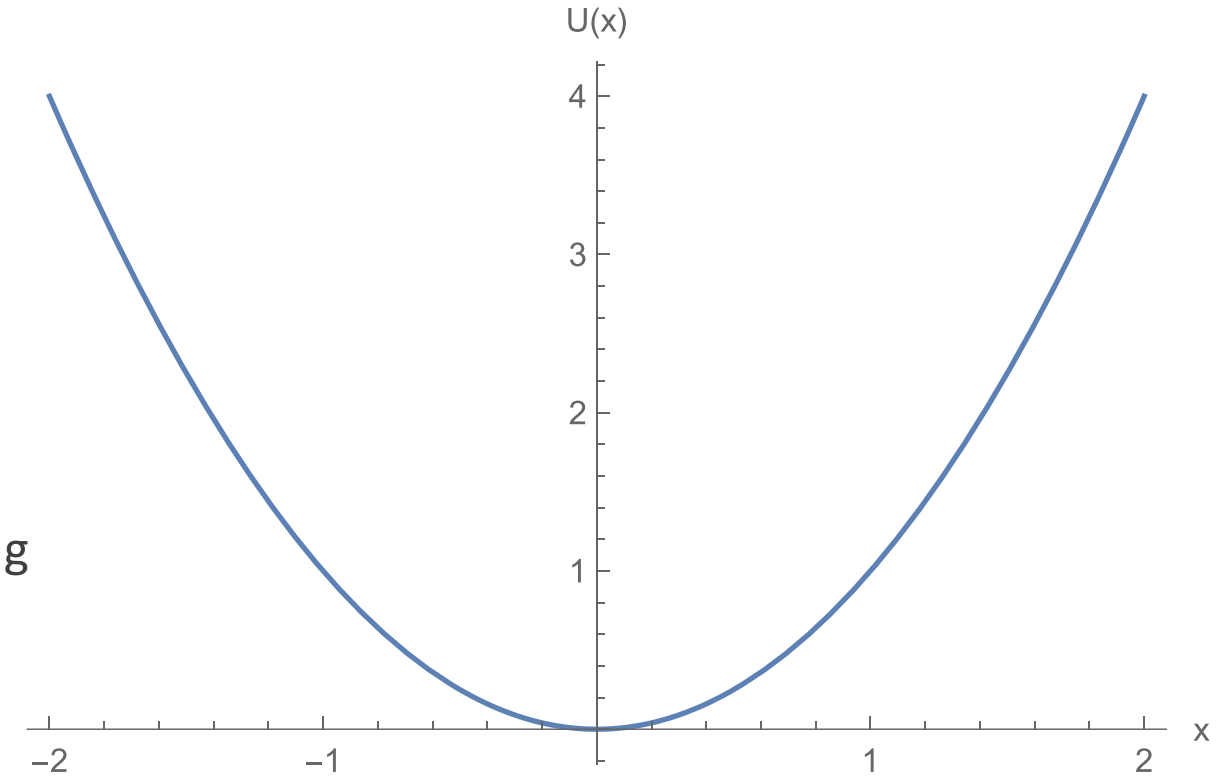
Quantum systems:

- Molecular vibrations
- Particles in optical traps
- Photons
- Sound (phonons)

Classically can imagine the system being a ball rolling up and down the potential (without friction)

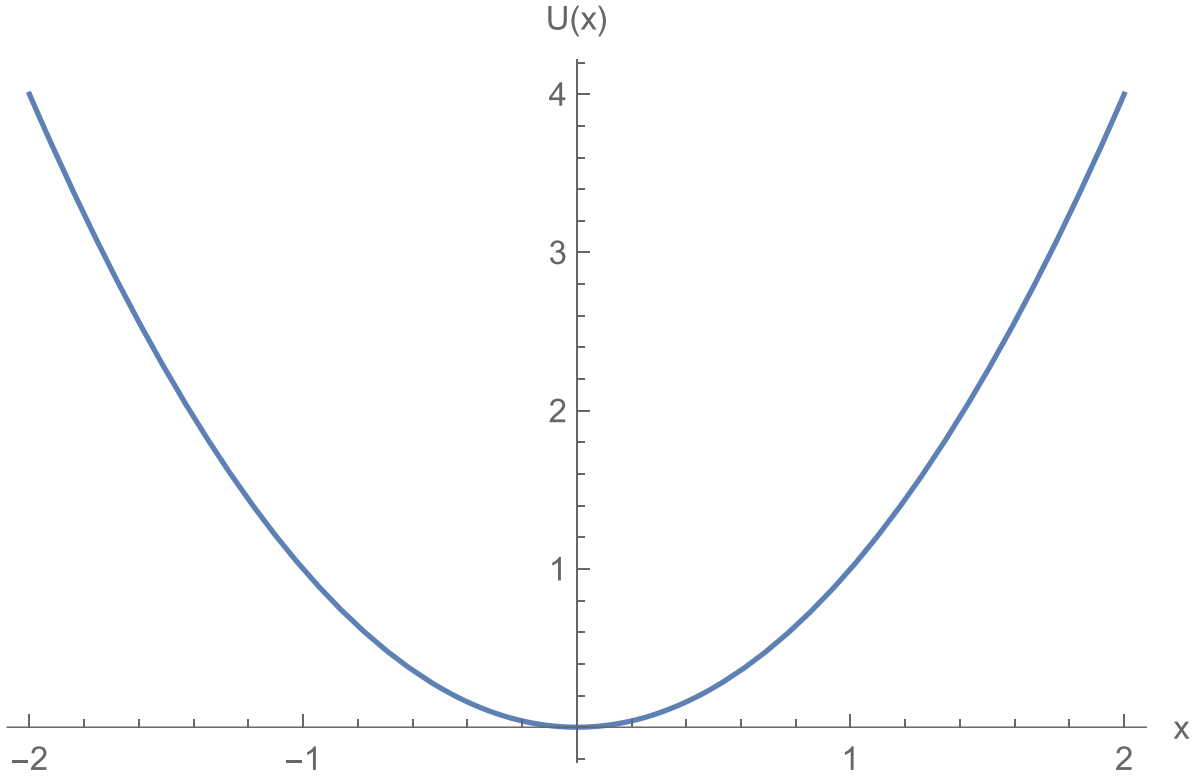
Classically forbidden to be in regions where:

$$U(x) > E$$



Harmonic Oscillator

Let’s solve this classically:



Harmonic Oscillator

Let’s solve this classically:

Harmonic Oscillator

Now quantum mechanically:

Harmonic Oscillator

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Higher states:

$$\psi_1(x) = A_1 e^{-\frac{x^2}{2b^2}}$$

$$\psi_2(x) = A_2 \frac{x}{b} e^{-\frac{x^2}{2b^2}}$$

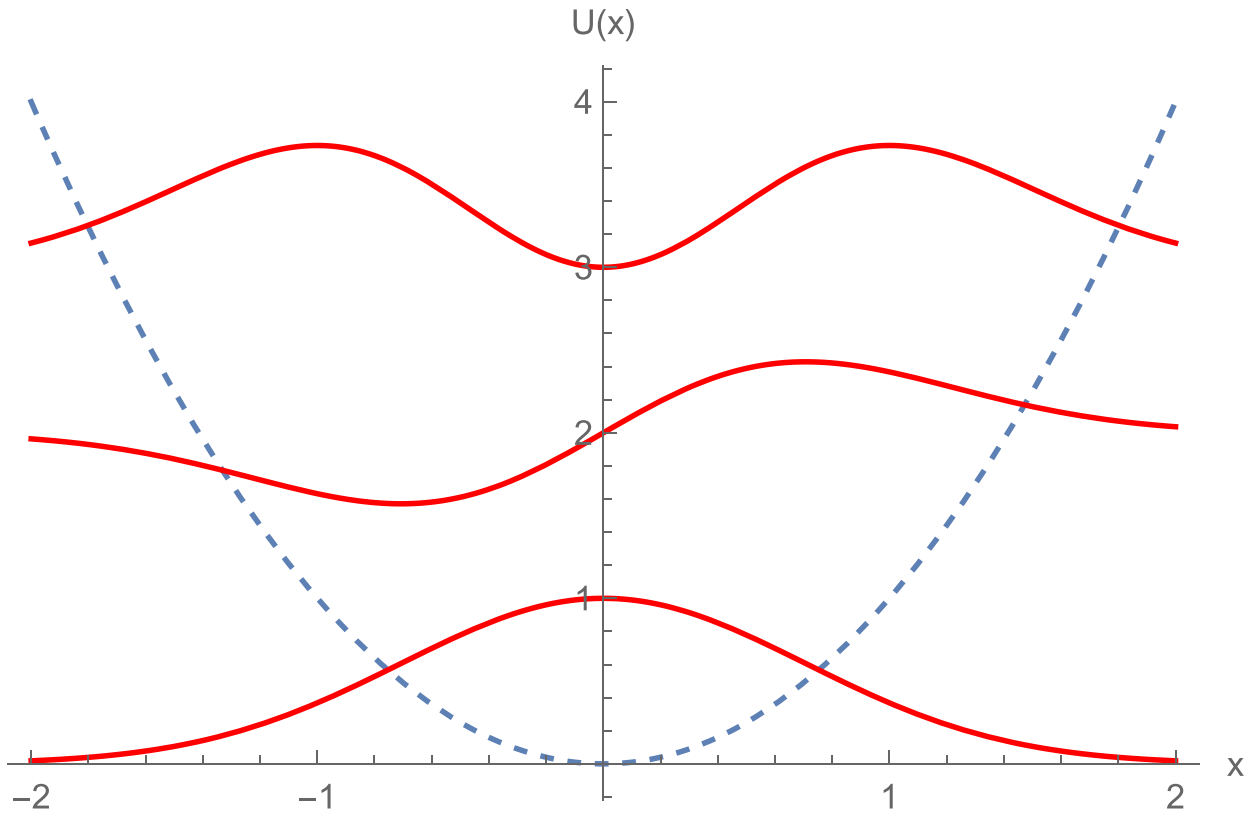
Hermite Polynomials

$$\psi_3(x) = A_3 \left(1 - \frac{2x^2}{b^2}\right) e^{-\frac{x^2}{2b^2}}$$

$$b = \sqrt{\frac{\hbar}{2\omega}}$$

Energies follow:

$$E_n = \left(n - \frac{1}{2}\right)\hbar\omega$$



Harmonic Oscillator

We can also use Heisenberg's uncertainty principle to derive the ground state energy:

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Harmonic Oscillator

Ground state energy can be determined by Heisenberg Uncertainty Principle

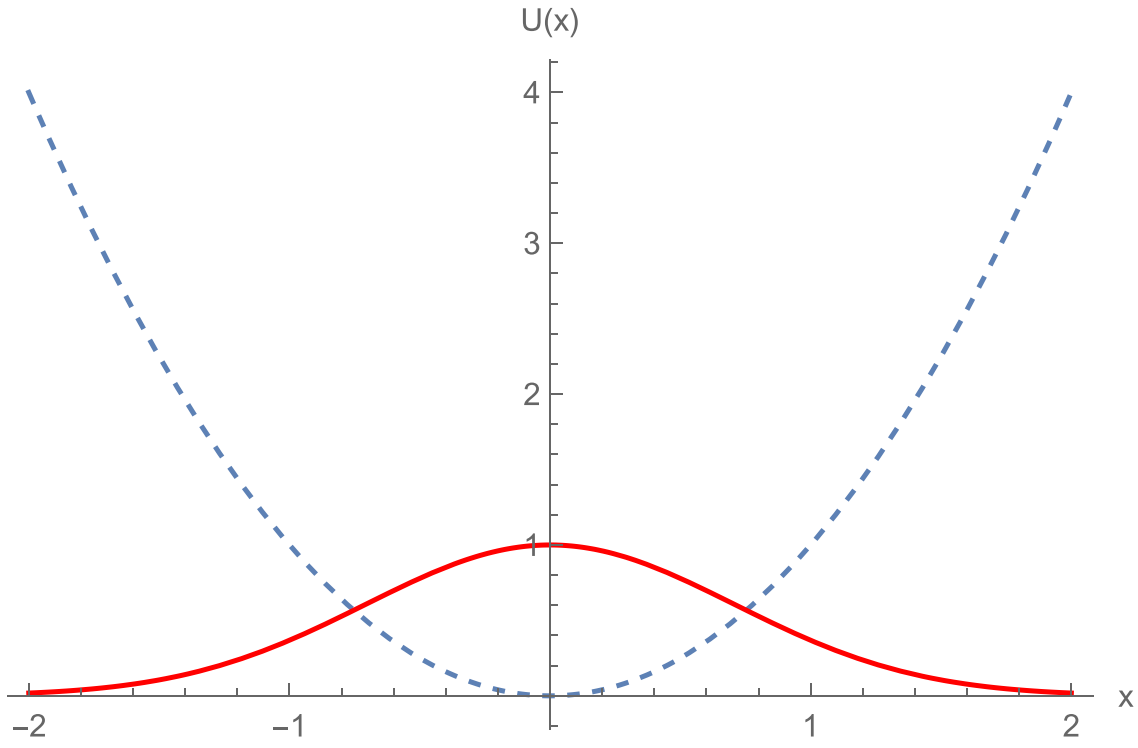
$$E_1 = \frac{1}{2}\hbar\omega$$

Since this is non-zero, a particle in a harmonic trap can never be stationary leading to zero-point motion

Lowest energy level restricted by Heisenberg uncertainty principle

This zero-point energy keeps liquid Helium from freezing at atmospheric pressures, even at absolute zero

True for any particle that is confined to a range of locations



Homework Questions

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