Phyx 320
Modern Physics

March 26, 2021

Reading: 40.5-40.8

Homework #9 and Reading Reflection Next Thursday 11:59 pm

Particle in a Box

Schrödinger Equation:

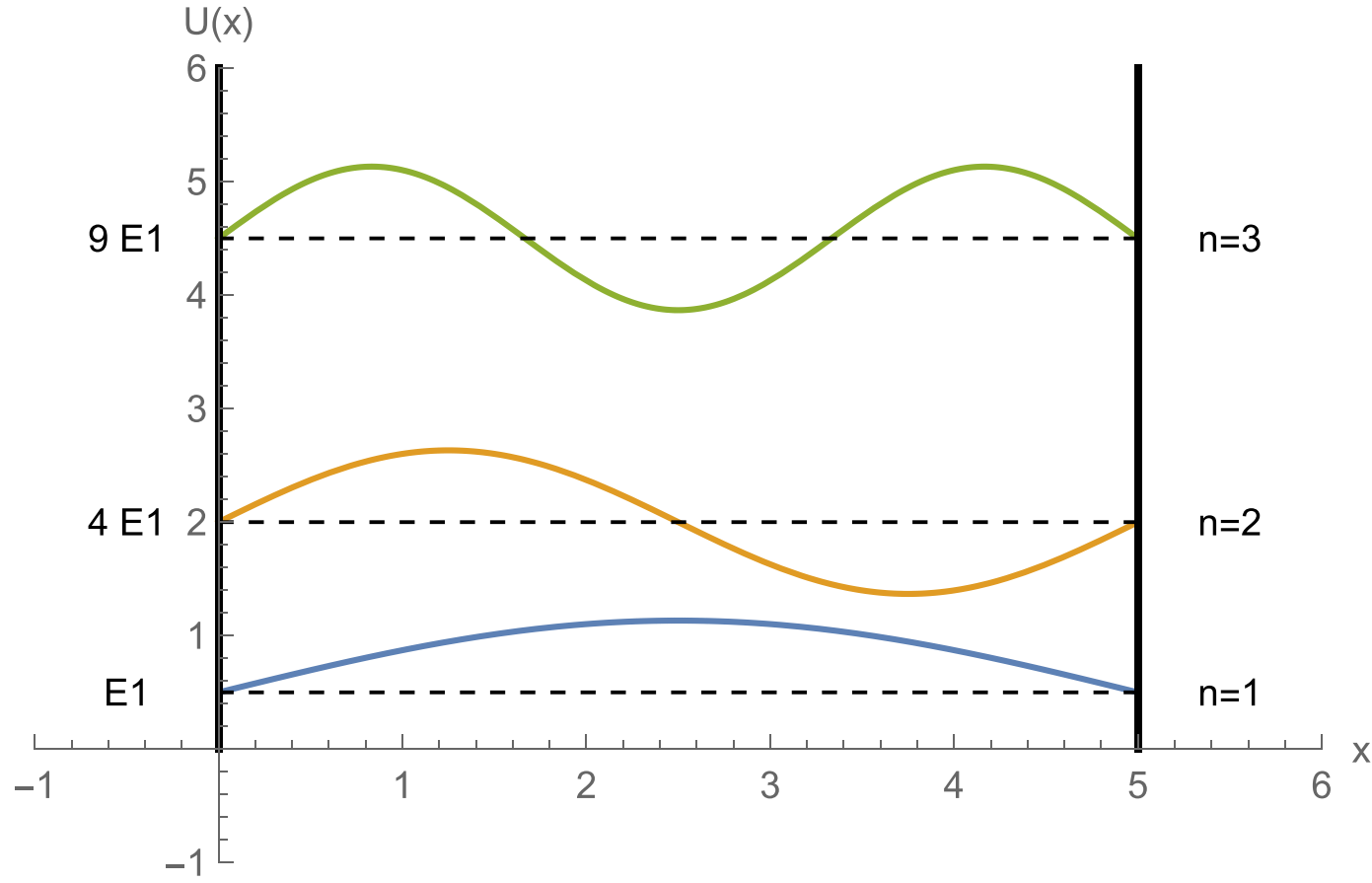
$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\,\psi(x)$$

Putting all this together:

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right); & 0 \le x \le L \\ 0; & x > 0 \; and \; x < L \end{cases}$$

Energies follow:

$$E = n^2\frac{h^2}{8mL^2}$$



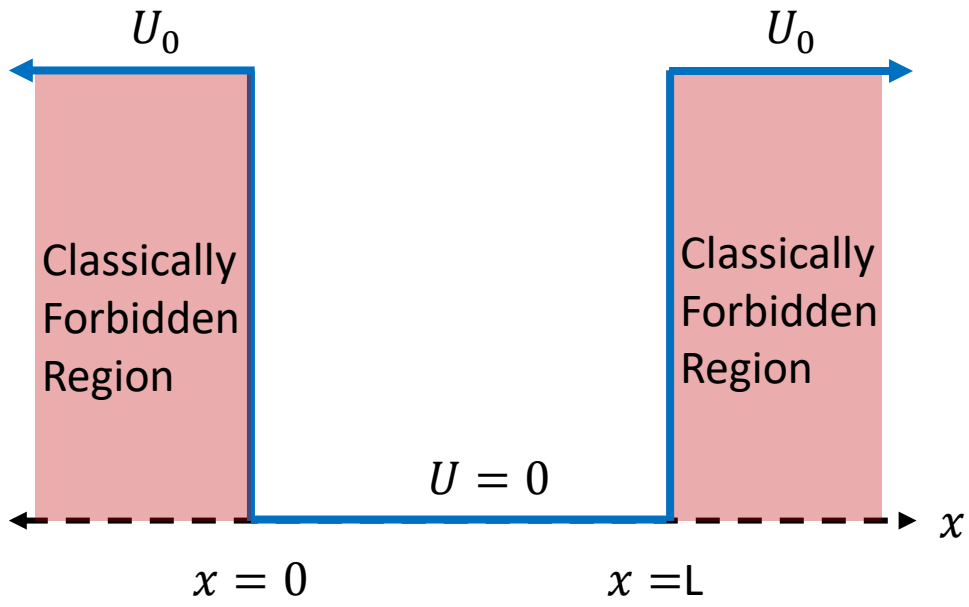
Finite Potential Wells

Previously, we studied the particle in a box which assumed infinity potential walls, but this would require infinity energy

More realistic models is potential with finite potential walls

Classically the particle is forbidden in any region where $E < U$

For $x < 0$ and $x > L$, classically forbidden for particles with $E < U_0$ but not for quantum mechanics



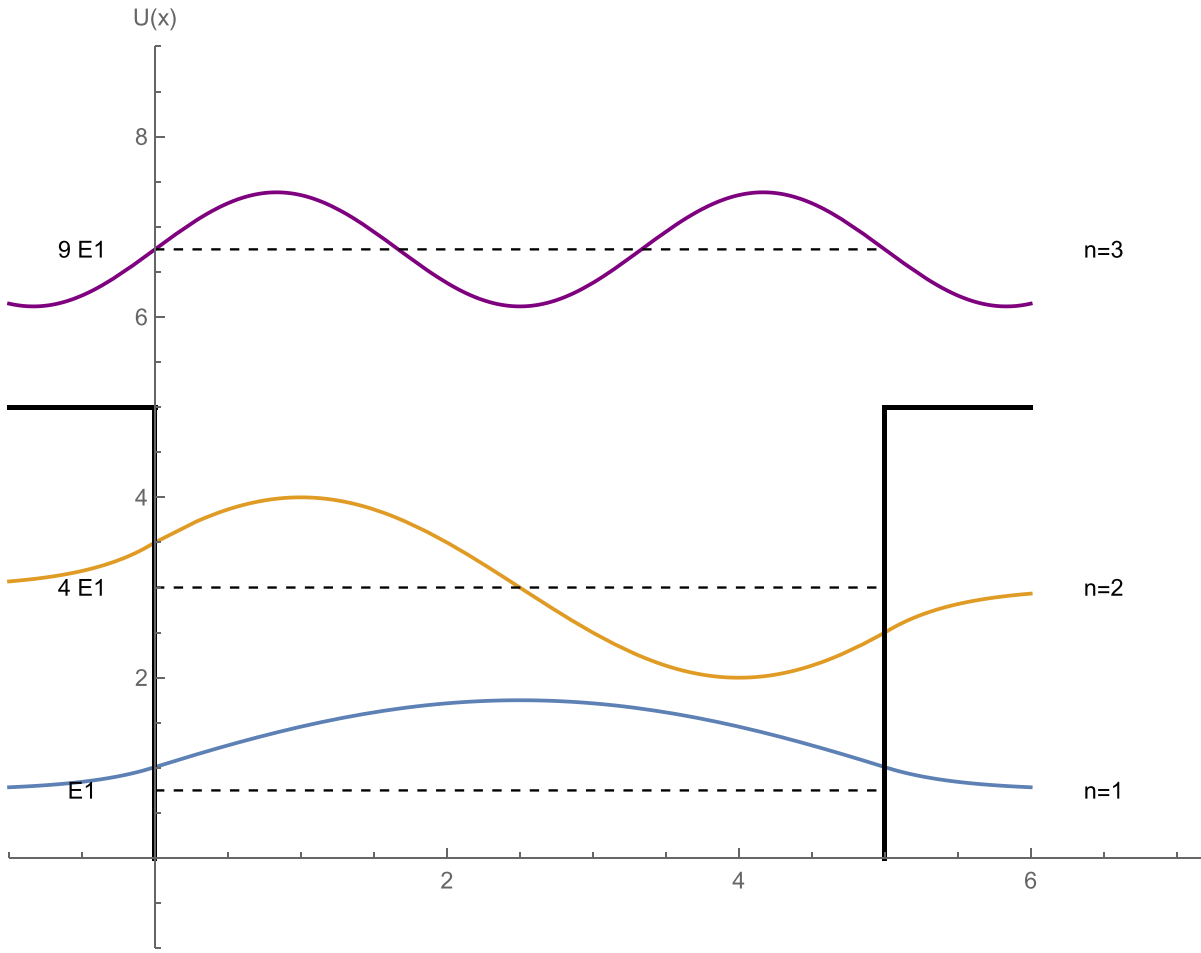
Finite Potential Wells

Since the potential is not infinite, the wavefunction extends into the walls

Inside well ($U = 0$): sine wave
Outside well ($U = U_0$): exponential

Two types of states:

- Bound states ($E < U_0$): quantized energies, finite number of states, most probability inside well but can leak into the classically forbidden regions
- Free states ($E > U_0$): non-quantized energies, infinite number of states, not constrained to be in well



Finite Potential Wells

Let’s focus in on the classically forbidden region:

Finite Potential Wells

Let’s focus in on the classically forbidden region:

Finite Potential Wells

Focusing on just $x \geq L$ region:

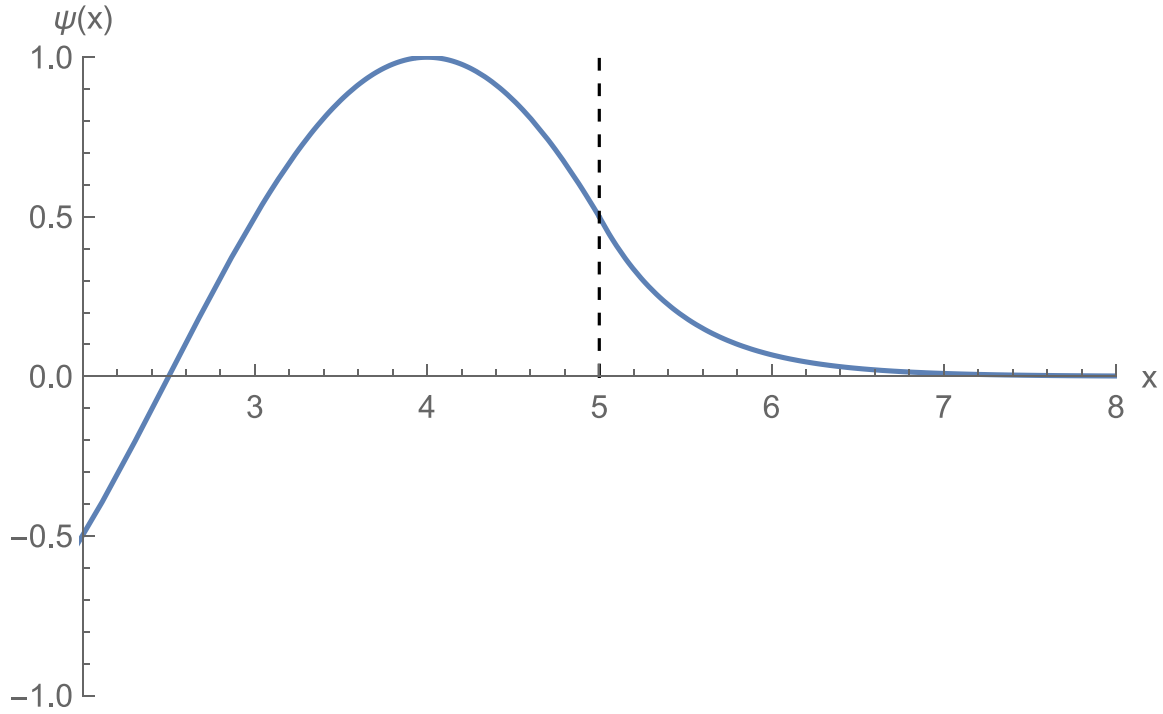
Finite Potential Wells

In classically forbidden region, wavefunction decays exponentially:

$$\psi(x) = \psi_{edge}\, e^{-\frac{x-L}{\eta}}$$

Wavefunction decays with a characteristic length scale, penetration depth:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$



Homework Questions

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