Phyx 320
Modern Physics

March 26, 2021

Reading: 40.5-40.8

Homework #9 and Reading Reflection Next Thursday 11:59 pm

Particle in a Box

Schrödinger Equation:

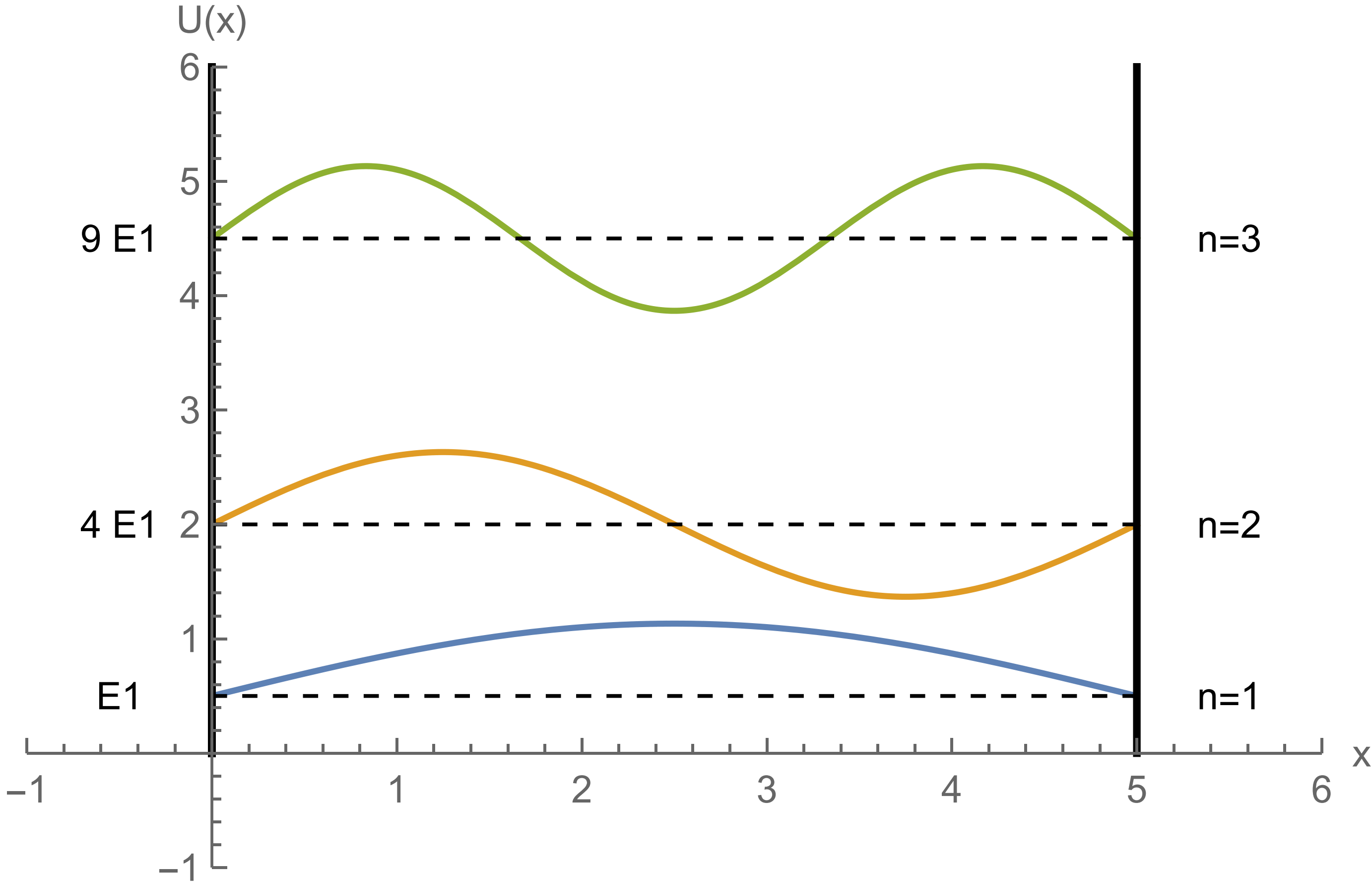
$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}[E - U(x)]\,\psi(x)$$

Putting all this together:

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right); & 0 \le x \le L \\ 0; & x > 0 \; and \; x < L \end{cases}$$

Energies follow:

$$E = n^2 \frac{h^2}{8mL^2}$$



Finite Potential Wells

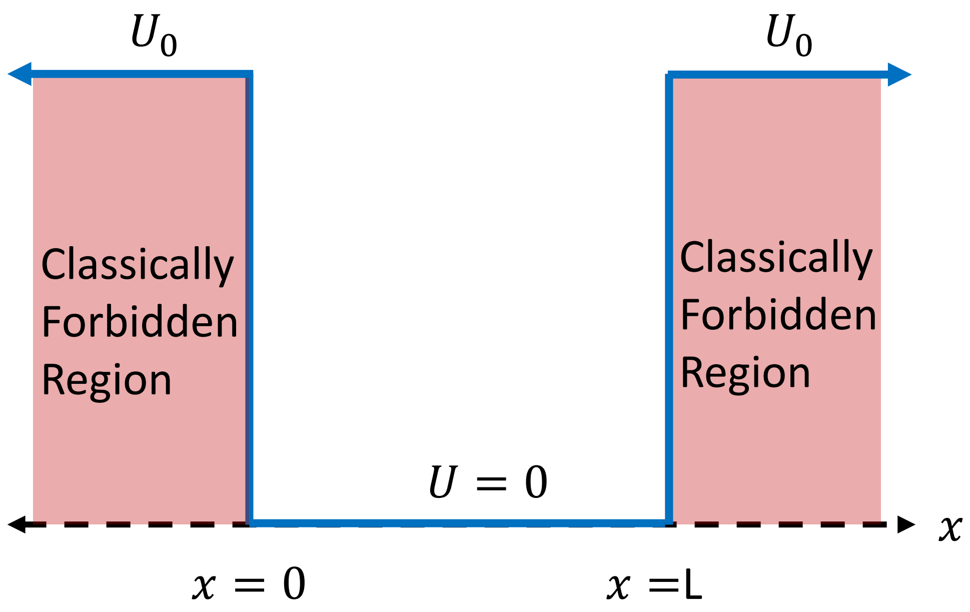
Previously, we studied the particle in a box which assumed infinity potential walls, but this would require infinity energy

More realistic models is potential with finite potential walls

Classically the particle is forbidden in any region where $E < U$

For $x < 0$ and $x > L$, classically forbidden for particles with $E < U_0$ but not for quantum mechanics

$E = k + U$



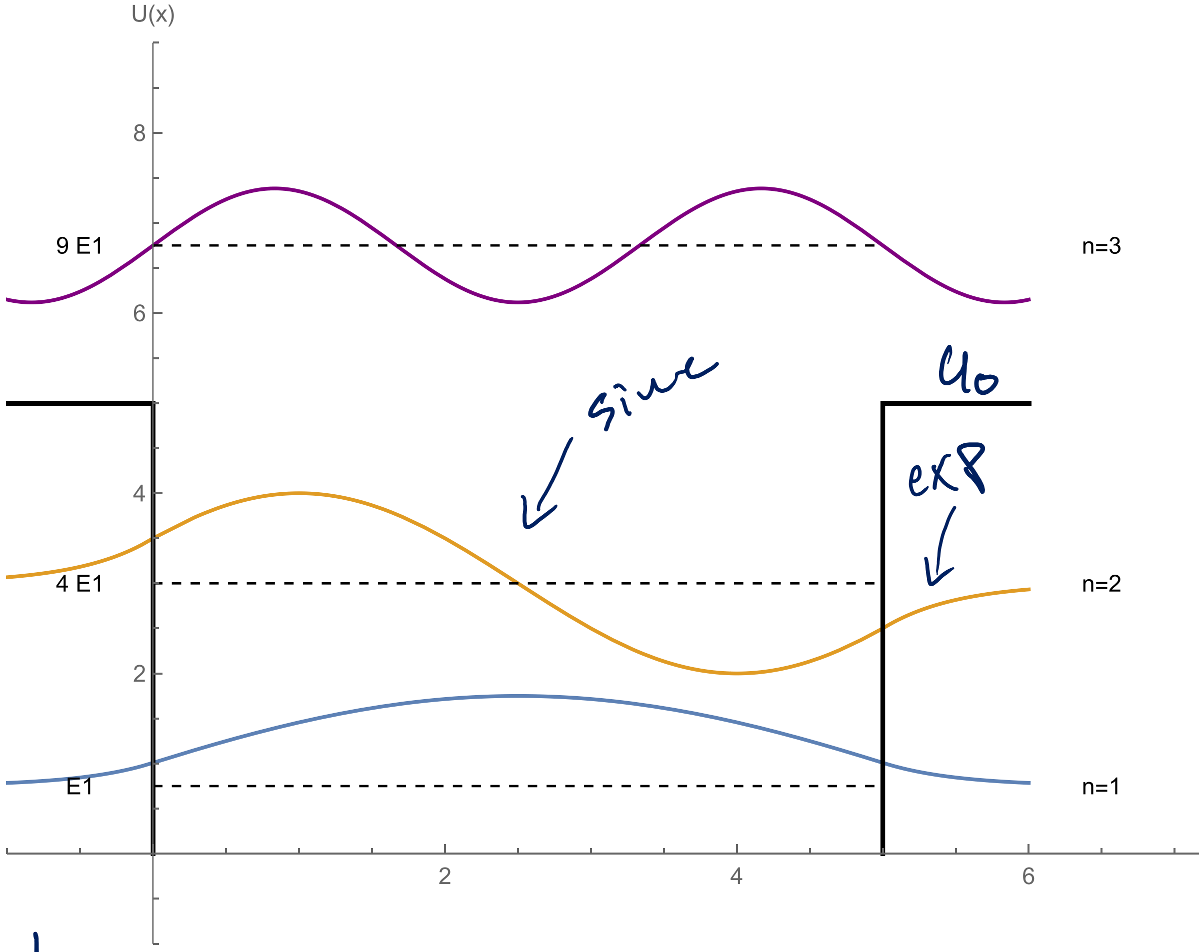
Finite Potential Wells

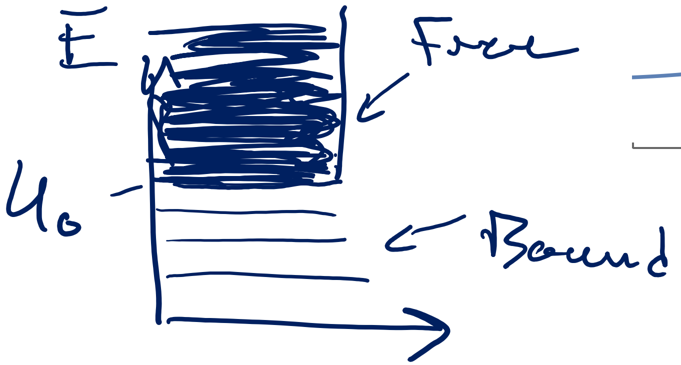
Since the potential is not infinite, the wavefunction extends into the walls

Inside well ($U = 0$): sine wave
Outside well ($U = U_0$): exponential

Two types of states:

- Bound states ($E < U_0$): quantized energies, finite number of states, most probability inside well but can leak into the classically forbidden regions
- Free states ($E > U_0$): non-quantized energies, infinite number of states, not constrained to be in well





Finite Potential Wells

Let's focus in on the classically forbidden region:

$$E < U_0 \Rightarrow E - U_0 < 0$$

$$\eta^2 = \frac{\hbar^2}{2m(U_0 - E)} > 0$$

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar}(E - U_0)\psi(x)$$

> 0

$$\frac{d^2\psi}{dx^2} = \frac{2m}{\hbar}(U_0 - E)\psi(x)$$

$$\frac{d^2\psi}{dx^2} = \frac{1}{\eta^2}\psi(x)$$

Finite Potential Wells

Let's focus in on the classically forbidden region:

Assume:

$$\psi_1(x) = e^{x/\eta}$$

$$\psi_2(x) = e^{-x/\eta}$$

$$\frac{d^2\psi_1}{dx^2} = \frac{1}{\eta}\left[\frac{1}{\eta} e^{x/\eta}\right] = \frac{1}{\eta^2} e^{x/\eta} = \frac{1}{\eta^2}\psi_1(x) \checkmark$$

$$\frac{d^2\psi_2}{dx^2} = -\frac{1}{\eta}\left[-\frac{1}{\eta} e^{x/\eta}\right] = \frac{1}{\eta^2} e^{-x/\eta} = \frac{1}{\eta^2}\psi_2(x) \checkmark$$

$$\psi(x) = A e^{x/\eta} + B e^{-x/\eta}$$

Finite Potential Wells

Focusing on just $x \geq L$ region:

$$\psi(x) = A e^{x/\eta} + B e^{-x/\eta}$$

$$x \to \infty, \psi \to 0$$

$$\lim_{x \to \infty} \psi(x) = 0 = A(\infty) + B(0)$$

$$A = 0$$

$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$

$$\psi(x) = B e^{-x/\eta}$$

ψ has to be continuous

$$\psi(x=L) = \psi_{edge} = B e^{-L/\eta}$$

$$B = \psi_{edge} e^{L/\eta}$$

$$\psi(x) = \psi_{edge} e^{-(x-L)/\eta}$$

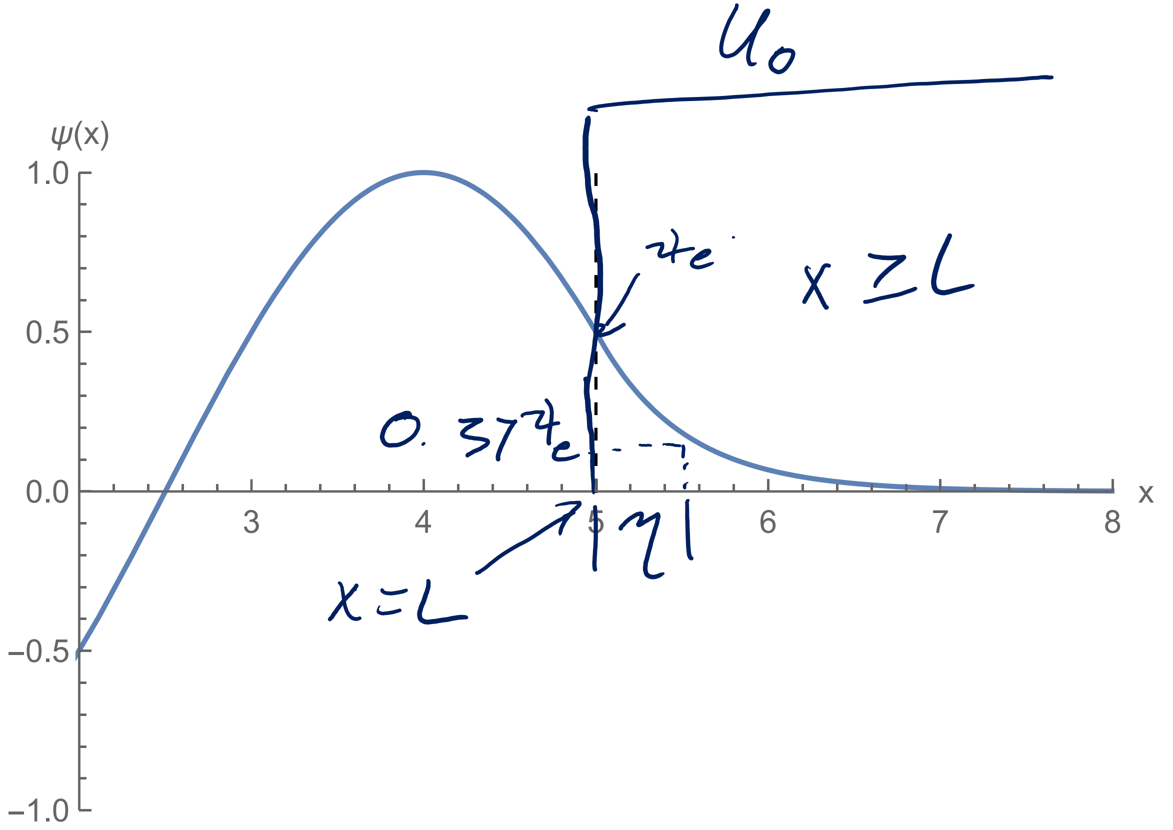
Finite Potential Wells

In classically forbidden region, wavefunction decays exponentially:

$$\psi(x) = \psi_{edge}\, e^{-\frac{x-L}{\eta}}$$

Wavefunction decays with a characteristic length scale, penetration depth:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$$



Homework Questions

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