Phyx 320
Modern Physics

March 19, 2021

Reading: 39.5 - 39.6, 40.1 - 40.4

Homework #8 and Reading Reflection Next Thursday 11:59 pm

Wavefunctions

Quantum particles (electrons, photons, protons, etc.) are described by wavefunctions $\psi(x)$

Wavefunctions follow superposition so can be added $\psi(x) = \psi_1(x) + \psi_2(x)$

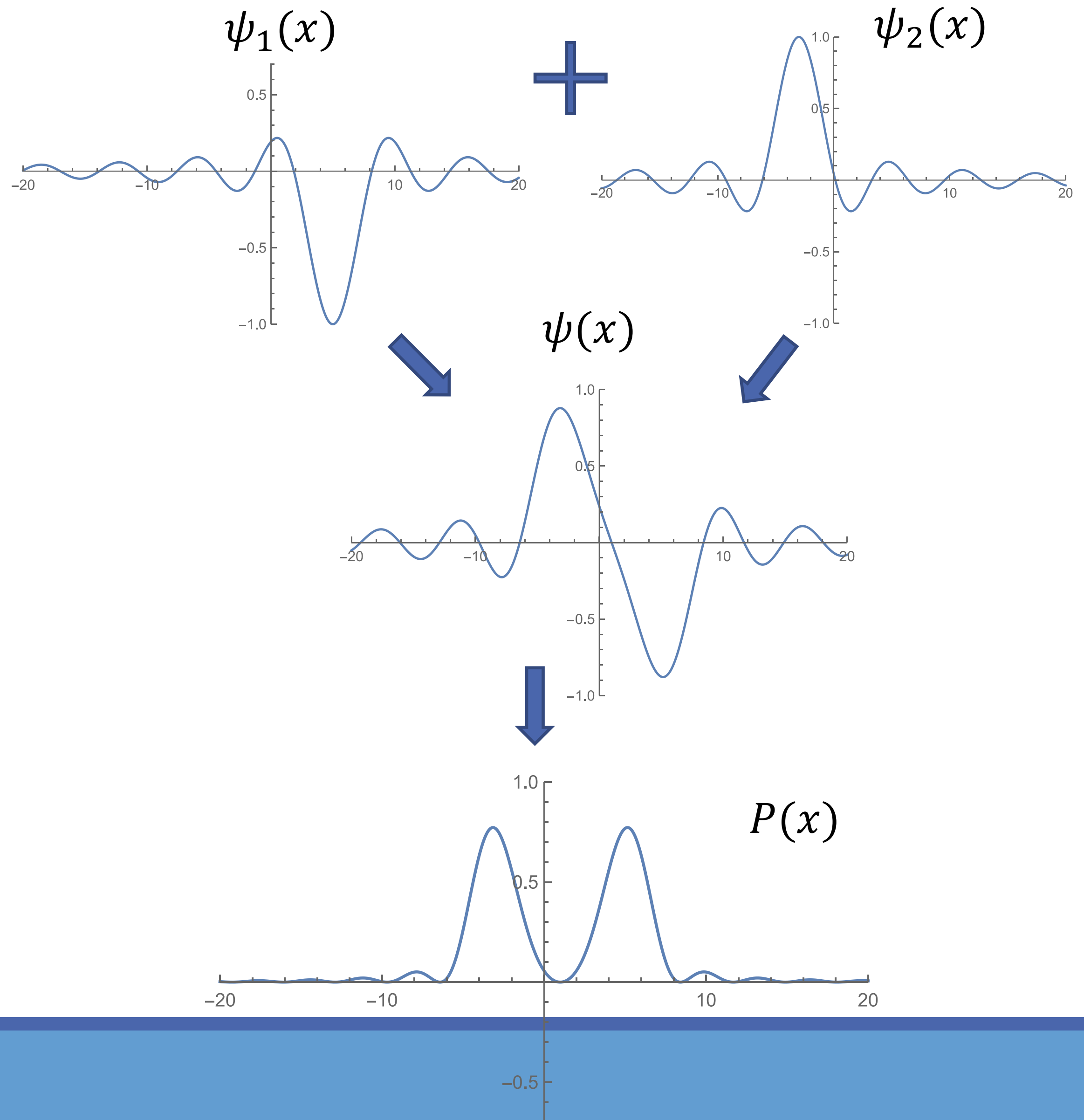
This produces interference effects

The probability density is defined to be

$$P(x) = |\psi(x)|^2$$

Wavefunctions must be normalized

$$\int_{-\infty}^{\infty} \left|\psi^2(x)\right| dx = 1$$



Wave Packets

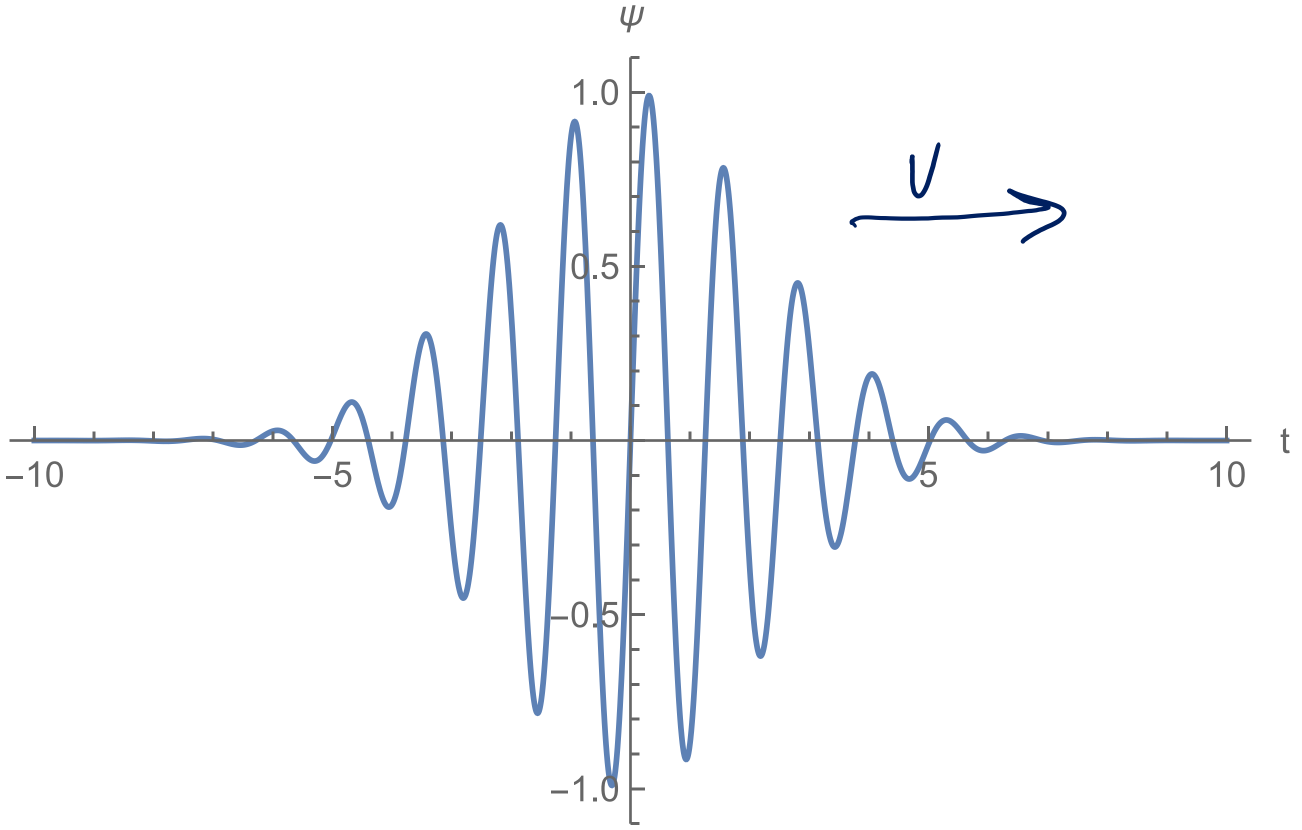
Although not a perfect model, wave packets are a useful way of thinking about wave-particle duality

Has both wave and particle characteristics

Travels at constant velocity, v

Has wavelength and can interfere and diffract like a wave

Is localized in space like a particle



Beat Notes

Wave packets are similar to beat notes

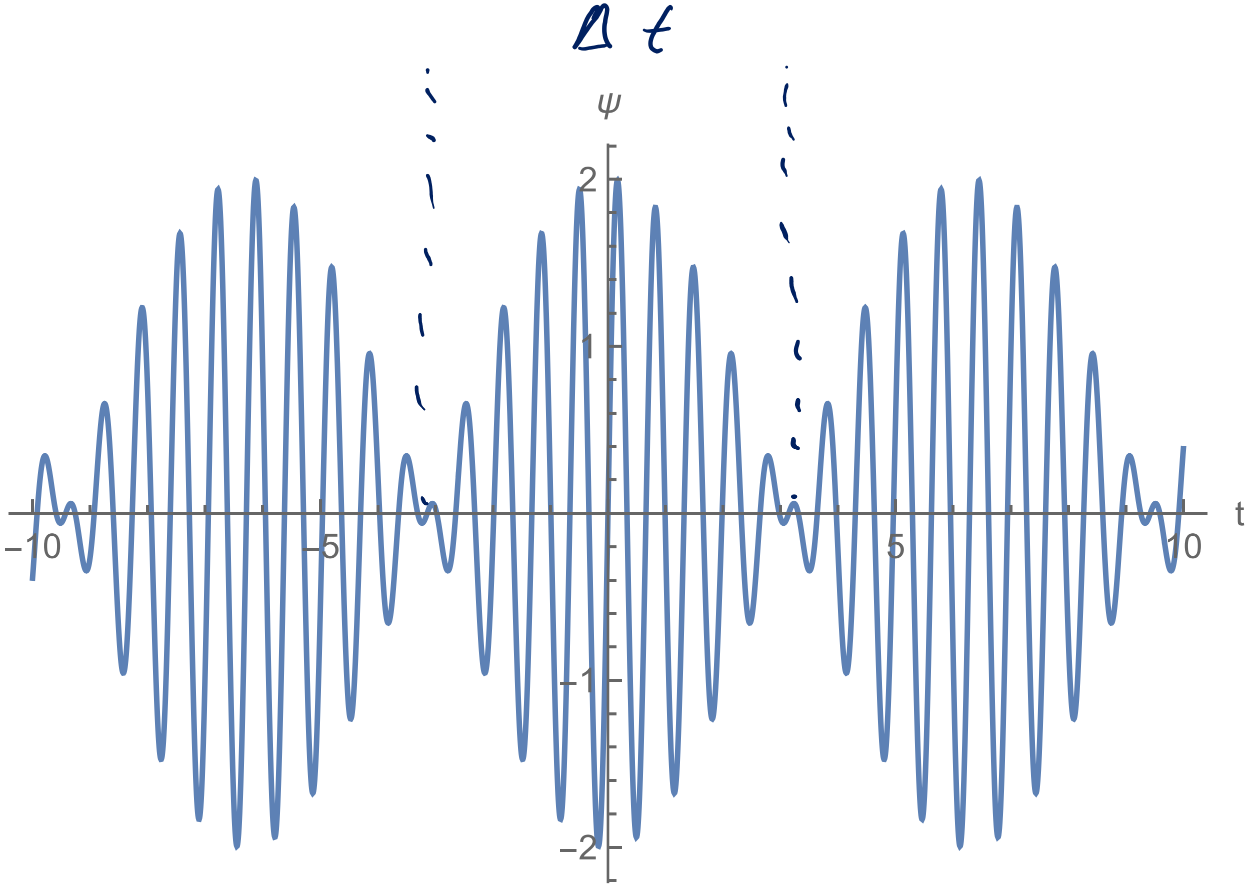
Beat notes are made when two waves with frequencies, f_1 and f_2, are added together and $f_1 \approx f_2$

$$F_{beat} = F_1 - F_2 = \Delta F$$

Duration:

$$\Delta t = \frac{1}{\Delta F}$$

$$\Delta F \Delta t = 1$$

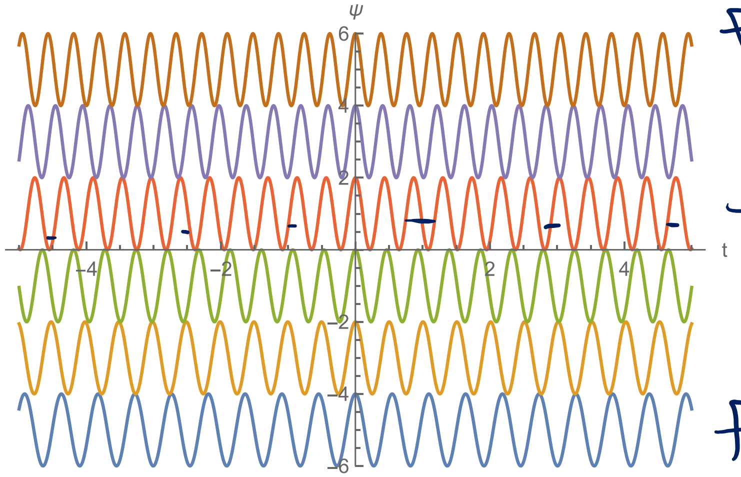


Wave Packets

We can extend this to describe wave packets but adding many waves together

Frequencies range from $f = f_0 - \frac{1}{2}\Delta f$ to $f = f_0 + \frac{1}{2}\Delta f$

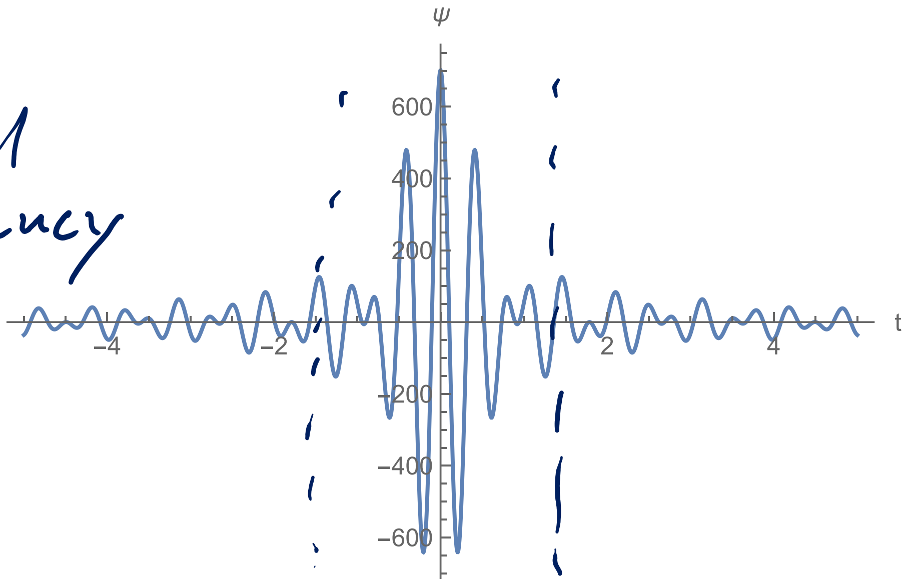
To make true wave packet we need infinite number of waves (Fourier series)



$f_0 + \frac{1}{2}\Delta f$

f_0

$f_0 - \frac{1}{2}\Delta f$



Δt

$\Delta f \Delta t \approx 1$

Frequency content

length

f_0 – central frequency

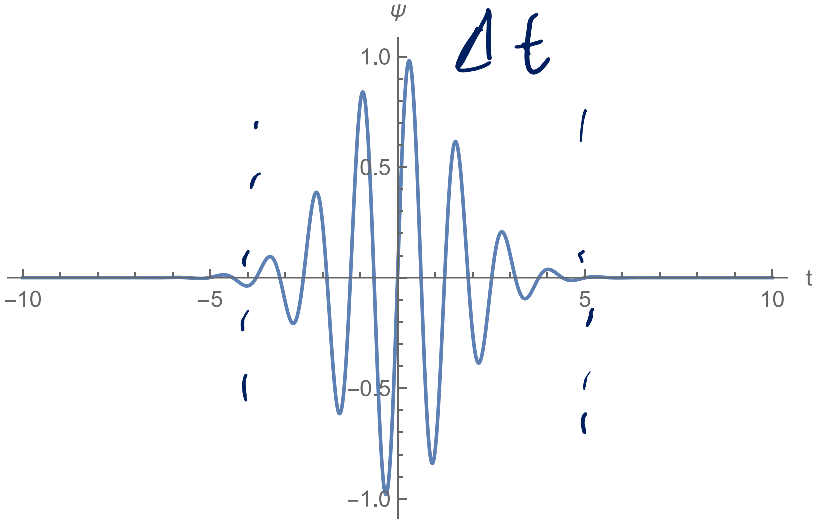
Wave Packets

If we only have the observed wave packet, then we can only know the frequency range that forms the wave packet

Can't know the exact central frequency

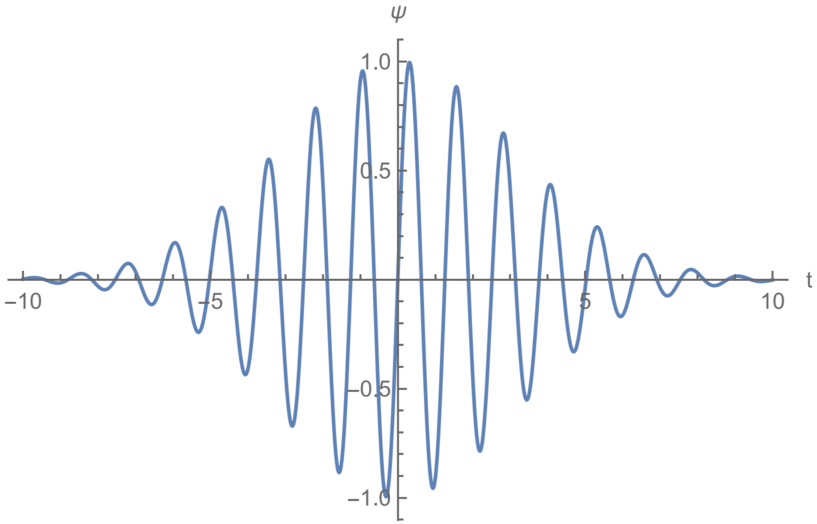
Longer packets allow to narrow that range

$$\Delta t = \frac{1}{\Delta F}$$



$$\Delta t \Delta F = 1$$

$\uparrow \quad \downarrow$



$$\Delta t \Delta F \geq 1$$

Heisenberg Uncertainty Principle

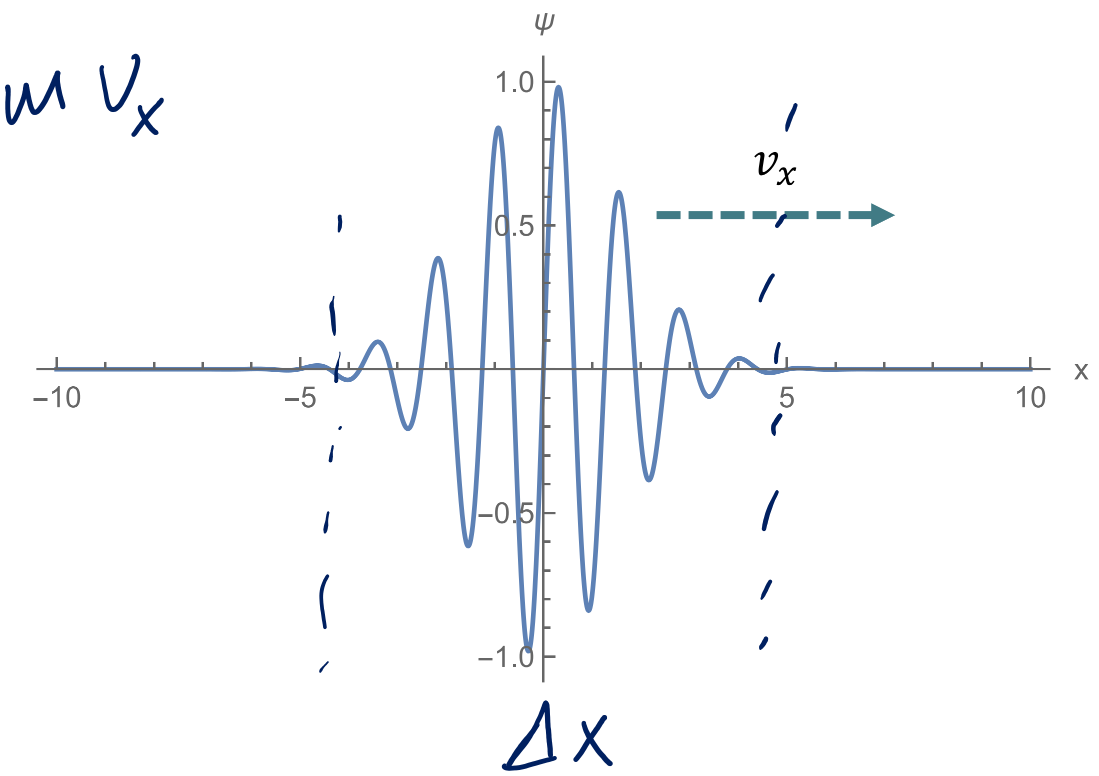
Now let's apply this to matter, with de Broglie wavelength $\lambda = h/p_x$

$p_x = m v_x$

Spatial length of wave packet:

$$\Delta x = v_x \Delta t = \frac{p_x}{m} \Delta t$$

$$\Delta t = \frac{m}{p_x} \Delta x$$



Heisenberg Uncertainty Principle

For waves

$$\lambda = \frac{h}{p_x}$$

$$F = \frac{v_x}{\lambda} = \frac{p_x}{m}\,\frac{p_x}{h} = \frac{p_x^2}{m h}$$

$$\frac{dF}{dp_x} = \frac{2 p_x}{m h} \approx \frac{\Delta F}{\Delta p_x}$$

$$\Delta F = \frac{2 p_x \Delta p_x}{m h}$$

$$\Delta t = \frac{m}{p_x} \Delta x$$

$$\Delta F \Delta t = \frac{2 \cancel{p_x} \Delta p_x}{\cancel{m} h} \, \frac{\cancel{m}}{\cancel{p_x}} \Delta x$$

$$= \frac{2}{h} \Delta p_x \Delta x$$

$$1 \le \Delta F \Delta t = \frac{2}{h} \Delta p_x \Delta x$$

$$\boxed{\Delta p_x \Delta x \ge \frac{h}{2}}$$

Heisenberg Uncertainty Principle

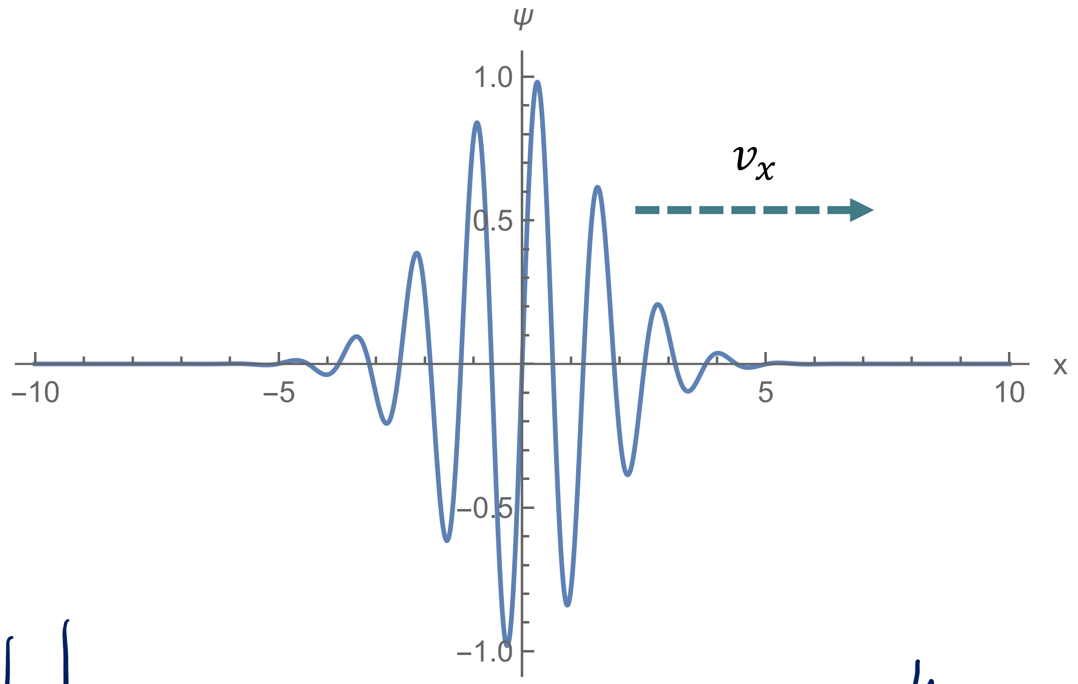
Heisenberg's Uncertainty Principle:

$$\Delta p_x \Delta x \geq \frac{h}{2}$$

We can't know both the momentum and the position of a particle at the same time

The uncertainties in measurements of position and momentum are anticorrelated
(more precise position = less precise momentum)

Because of the wave nature of matter, the properties of a particle are inherently uncertain



$= \Delta x$

$$\Delta E \Delta t \geq \frac{h}{2}$$

$$\lambda = \frac{h}{p}$$

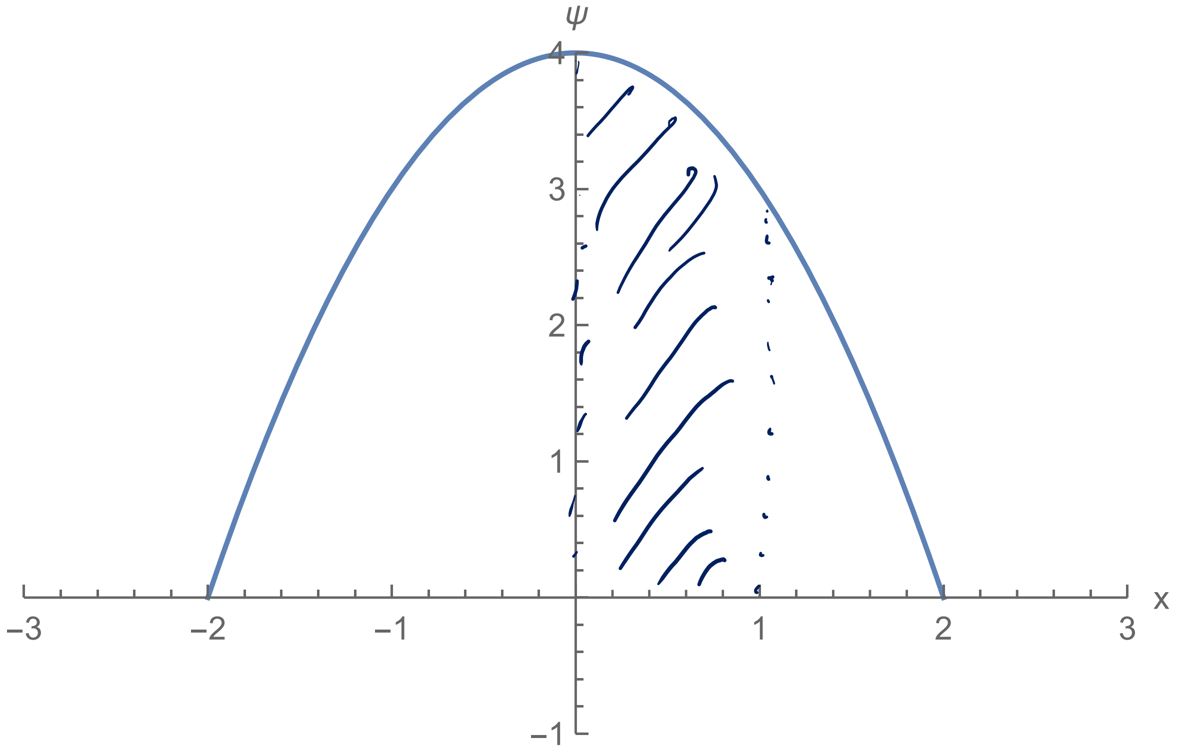
$\Delta\lambda_1$ $\Delta\lambda_2$ $\Delta\lambda_2 >> \Delta\lambda_2$

Quiz 7

Let's say we have a wavefunction ψ(x) = 4-x^2 which is constrained to be in the range x=(-2, 2).

1. What constant is needed to normalize this wavefunction? (Hint since the wavefunction is only defined in the range x=(-2, 2) the normalization is not done with (-inf, inf))

2. What is the probability of finding the particle described by this wavefunction between x=(0,1)?



Quiz 7

1) $$1 = \int_{-2}^{2} |k\psi(x)|^2 \, dx$$

$$\psi(x) = 4 - x^2$$

$$\Downarrow$$

$$\psi(x) = \sqrt{\frac{15}{512}}\left(4 - x^2\right)$$

$$1 = k^2 \int_{-2}^{2} \left(16 - 8x^2 + x^4\right) dx$$

$$1 = k^2 \left[16x - \frac{8}{3}x^2 + \frac{1}{5}x^5\right]\Bigg|_{-2}^{2}$$

$$1 = 2k^2\left[32 - \frac{64}{3} + \frac{32}{5}\right]$$

$$1 = k^2 \, \frac{512}{15} \implies k = \sqrt{\frac{15}{512}}$$

Quiz 7

$$2) \quad P_{x=(0,1)} = \int_0^1 \left[\sqrt{\frac{15}{512}}\,(4-x^2)\right]^2 dx$$

$$= \frac{15}{512}\int_0^1 (16-8x^2+x^4)\,dx$$

$$= \frac{15}{512}\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]\Big|_0^1$$

$$= \frac{15}{512}\left(16-\frac{8}{3}+\frac{1}{5}\right) = \frac{203}{512} \simeq 40\%$$

Homework Questions

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