Phyx 320
Modern Physics

March 15, 2021

Reading: 39.1 – 39.4

Homework #7 and Reading Reflection Thursday 11:59 pm

Probability Density

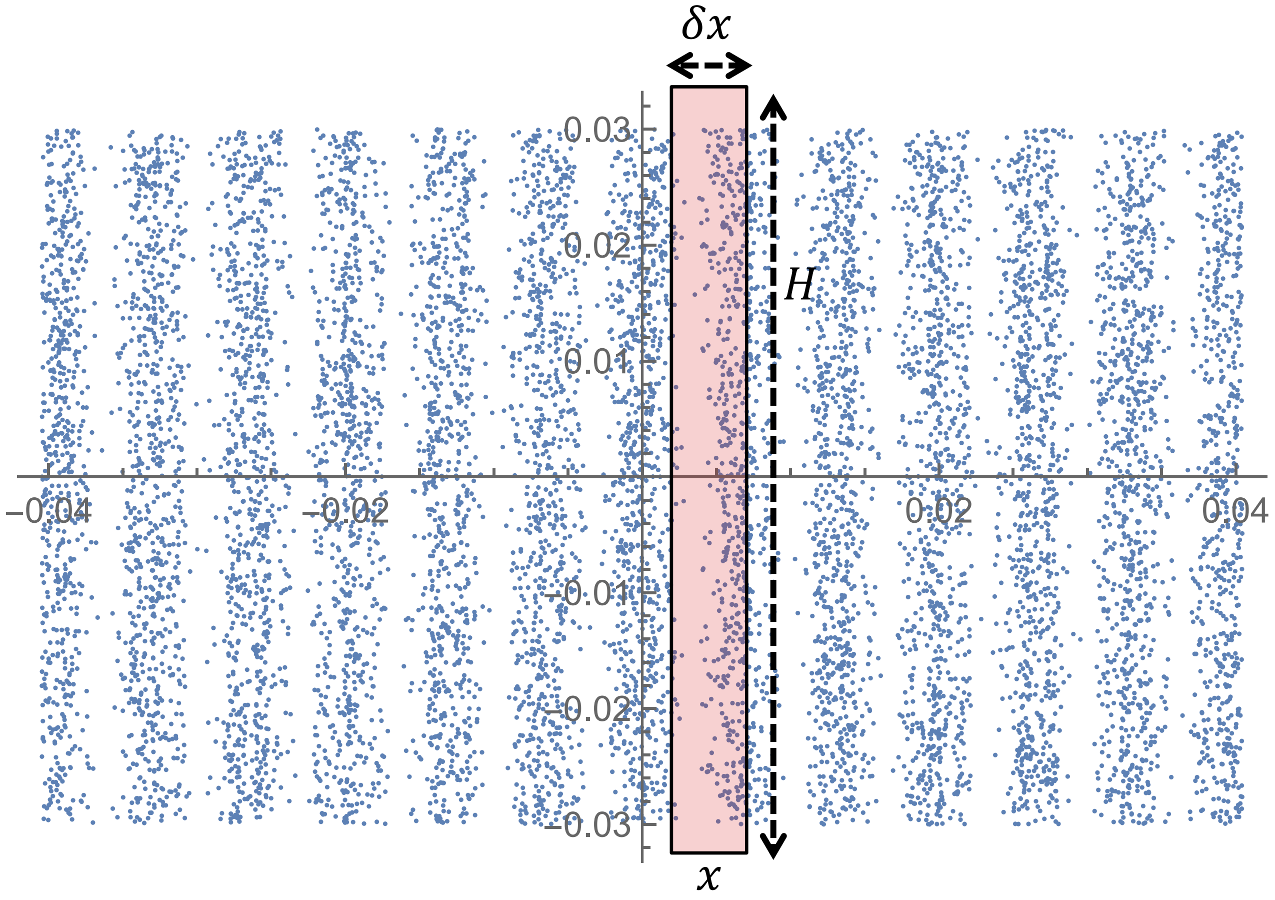
The probability density for photons is directly proportional to the square of the amplitude of the corresponding electromagnetic wave

$$P(x) \propto A(x)^2 \cancel{\propto} I(x)$$

Next lecture we will discuss the wavefunction which is generalization of this to every quantum particle

$$P(x) \propto |\psi(x)|^2$$

Wavefunction
(complex number)



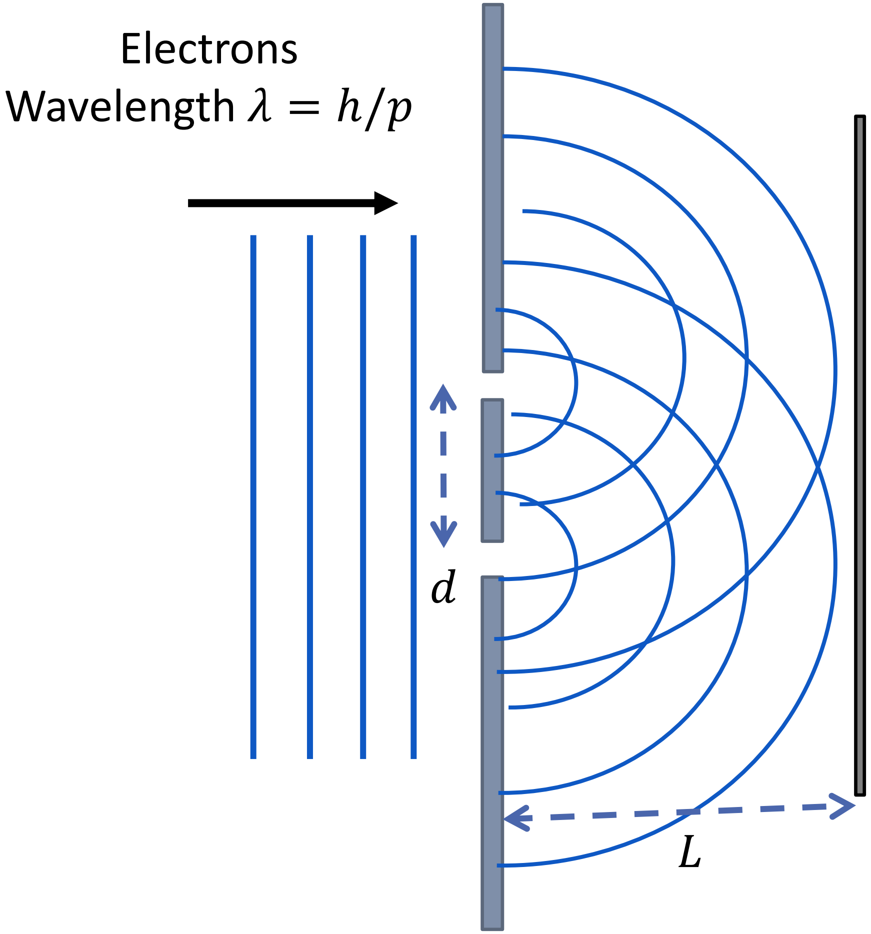
Wave Function

To extend our discussion of the double-slit experiment to electrons, we assert that electrons are described by a wavefunction $\psi(x)$

The probability density is then the wavefunction squared

$$P(x) = |\psi(x)|^2$$

Wavefunction
(complex number)



Wave Function

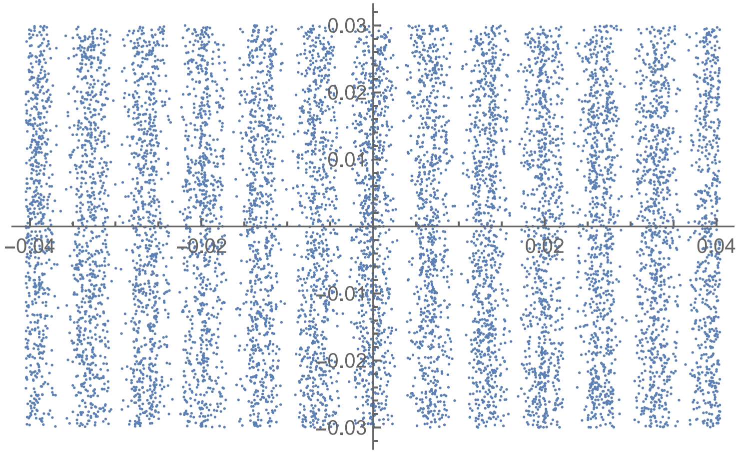
We can work backwards from the observed pattern of electrons to the probability density, $P(x)$

Where $P(x)$ has a maximum are locations more likely to find an electron

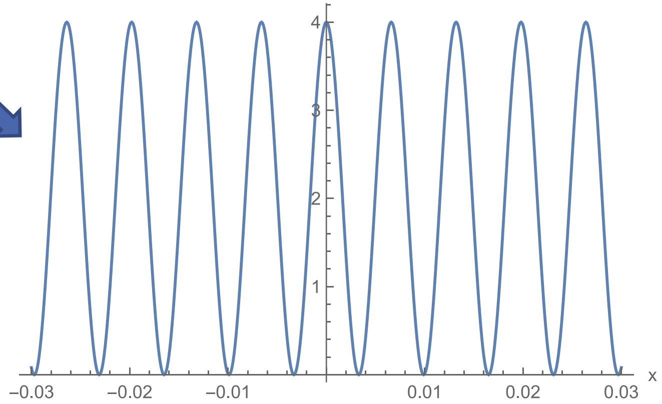
With many electrons going through the slits, more probability = more electrons

Wavefunction, $\psi(x)$, is square root of $P(x)$

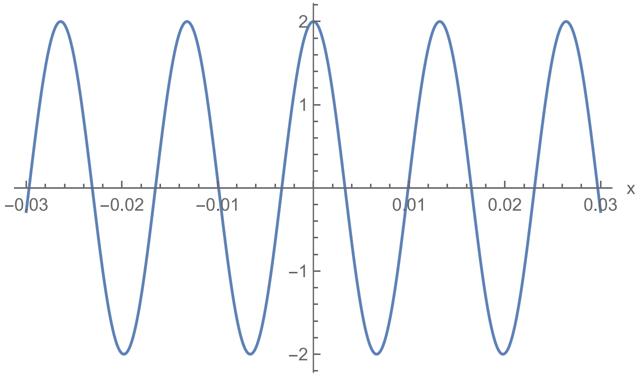
We can't observe $\psi(x)$ directly so any function that yields the observed $P(x)$ is a valid wavefunction ($\psi(x) = -\psi(x)$)



$P(x) = |\psi(x)|^2$



$\psi(x)$



Superposition

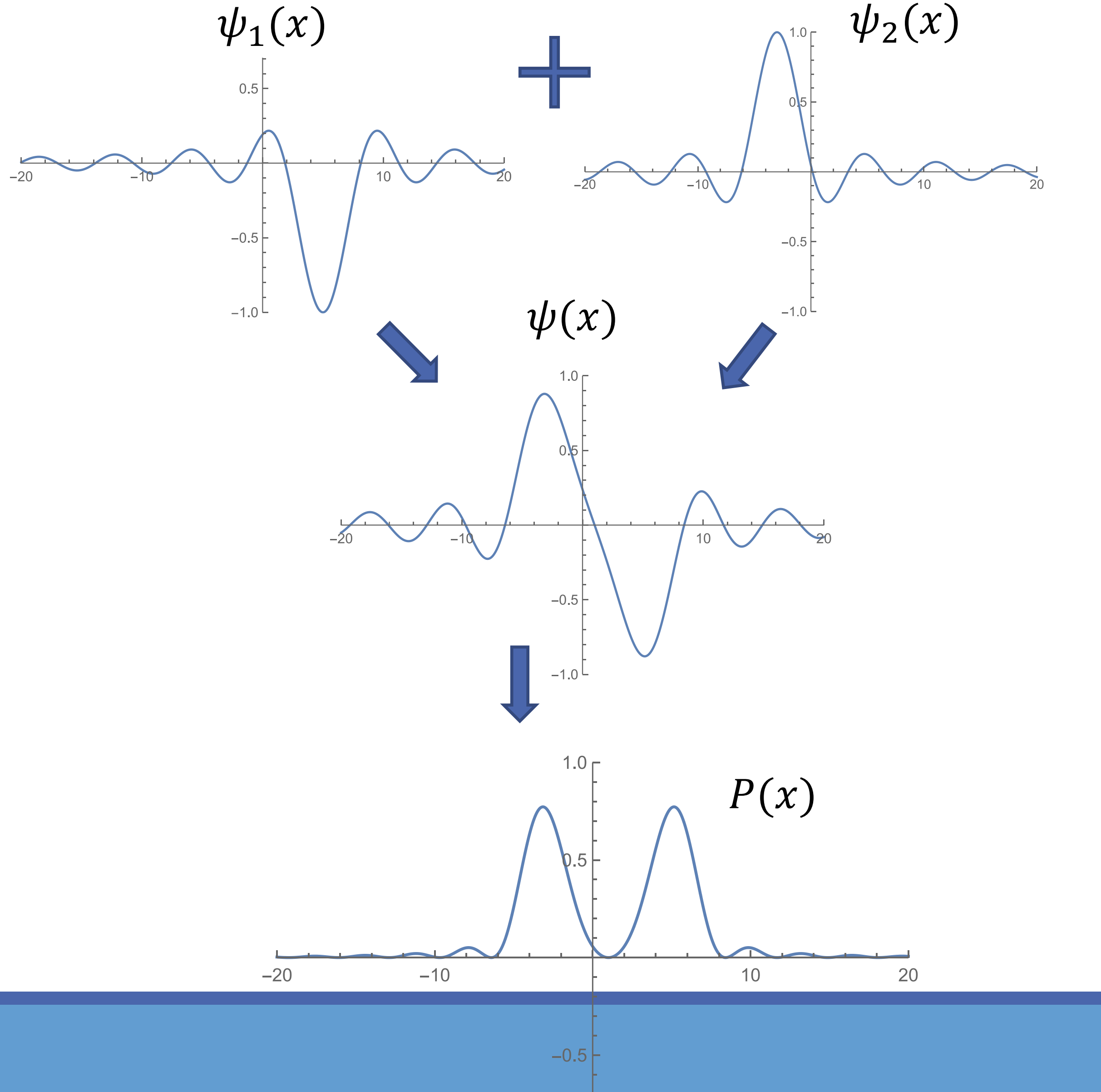
Wavefunctions add together just like waves

Quantum particles can be a superposition of multiple states

Let's say we have some system where the particle has equal likelihood of being in two states, $\psi_1\ (x)$ or $\psi_2\ (x)$

$$\psi(x) = \psi_1(x) + \psi_2(x)$$

For example, the electrons in the double-slit experiment are in a superposition of going through slit 1 and slit 2



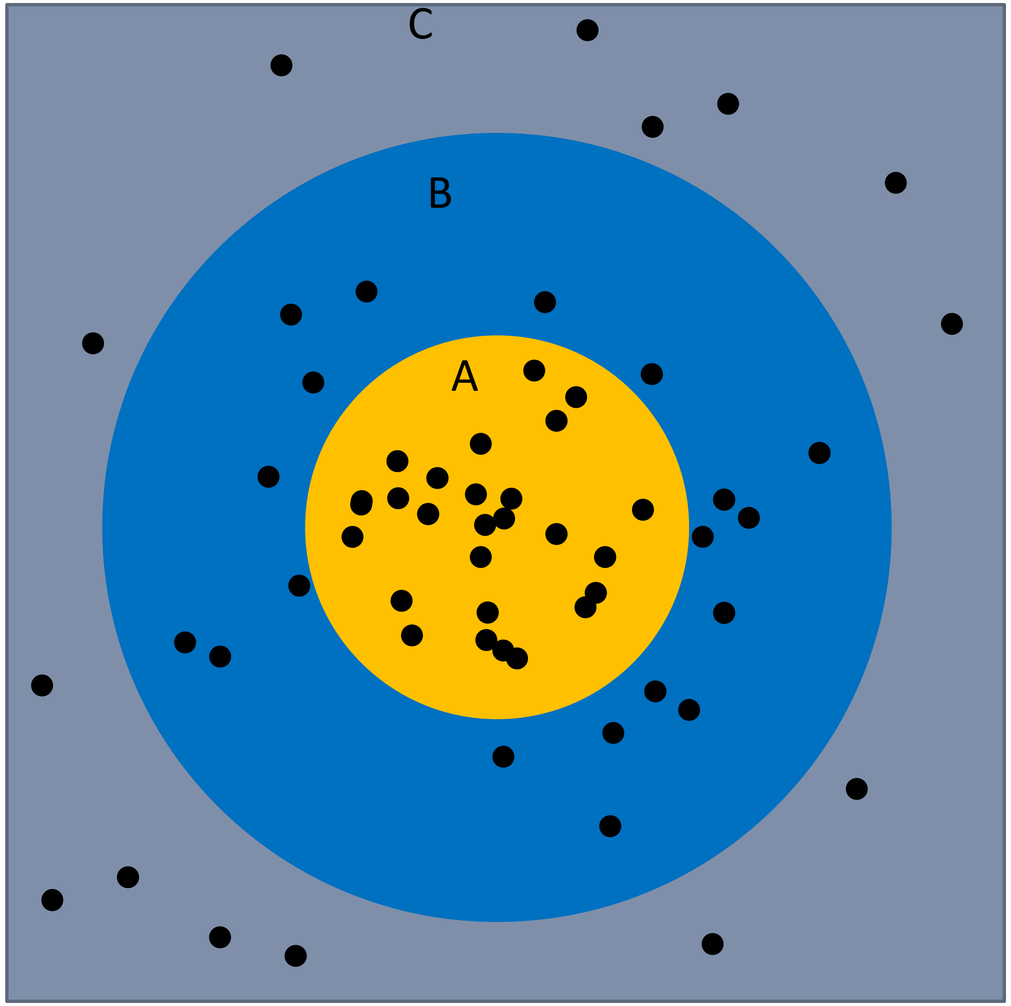
Normalization

In order to make logical sense, we have to force the probability of the particle landing somewhere to be 100%

Going back to dart board example

$$Prob_A + Prob_B + Prob_C = 1$$

$$\sum_{\text{all } i} P_i = 1$$



Normalization

Wavefunctions are continuous so we must integrate

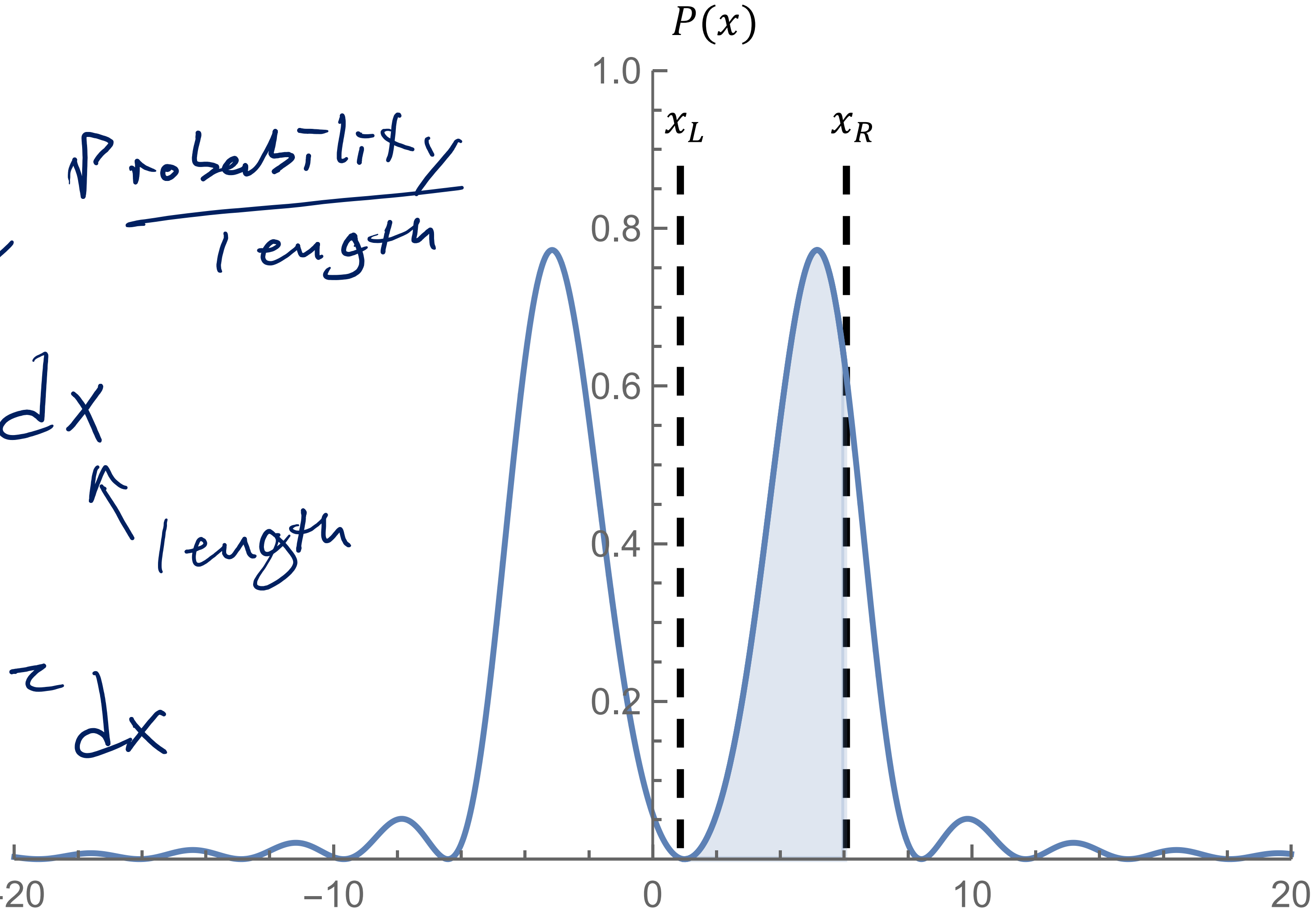
Probability of finding particle in range (x_L, x_R)

$$\frac{\text{Probability}}{\text{length}}$$

$$\text{Prob}_{(x_L,x_R)} = \int_{x_L}^{x_R} P(x)\, dx$$

length

$$= \int_{x_L}^{x_R} |\psi(x)|^2 dx$$



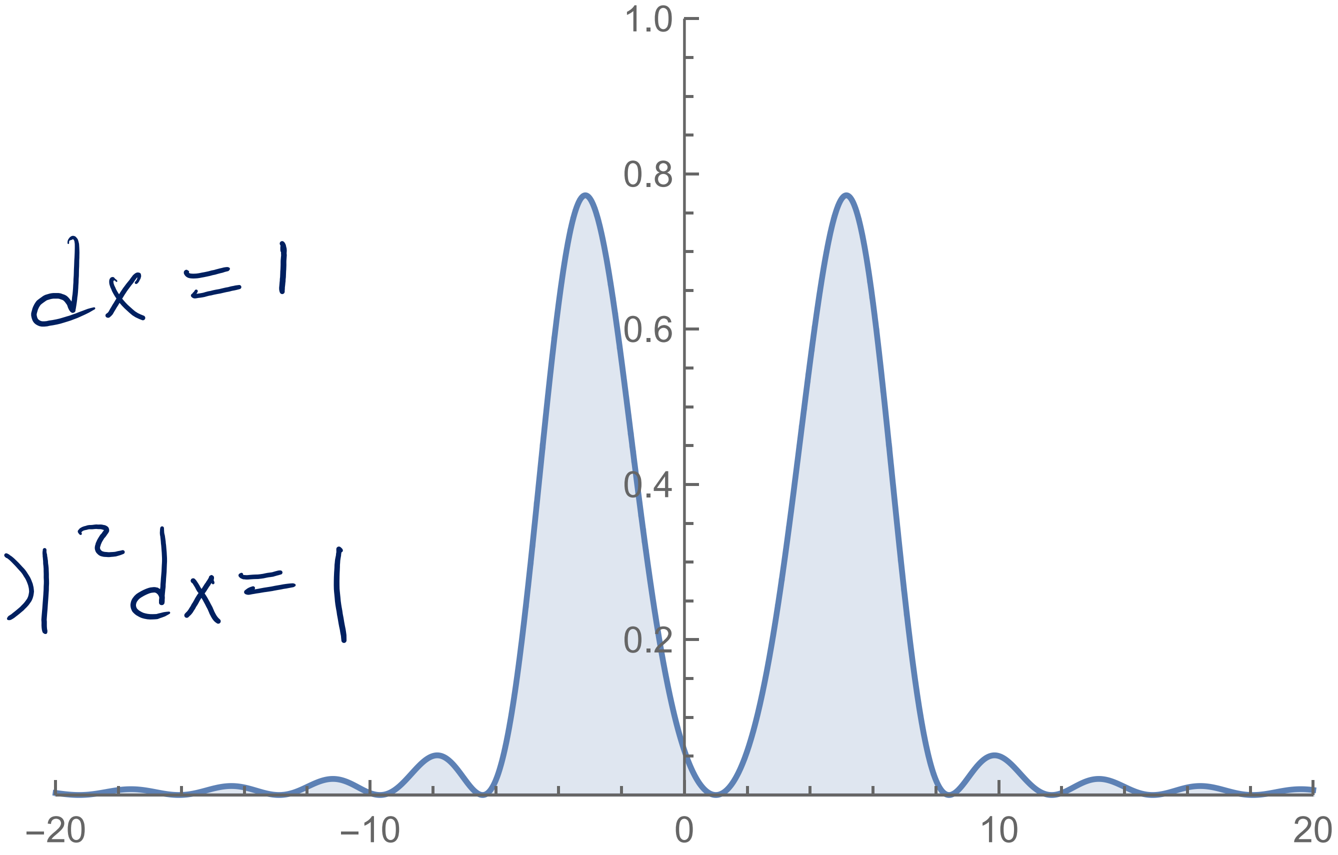
Normalization

If we integrate $P(x)$ over all allowed values of x: $(-\infty, \infty)$, we must get 1

This is the same as saying the particle must be somewhere

$$\text{Prob}_{(-\infty,\infty)} = \int_{-\infty}^{\infty} P(x)\, dx = 1$$

$$= \int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$$



Normalization

What if we have a wavefunction that isn't normalized?

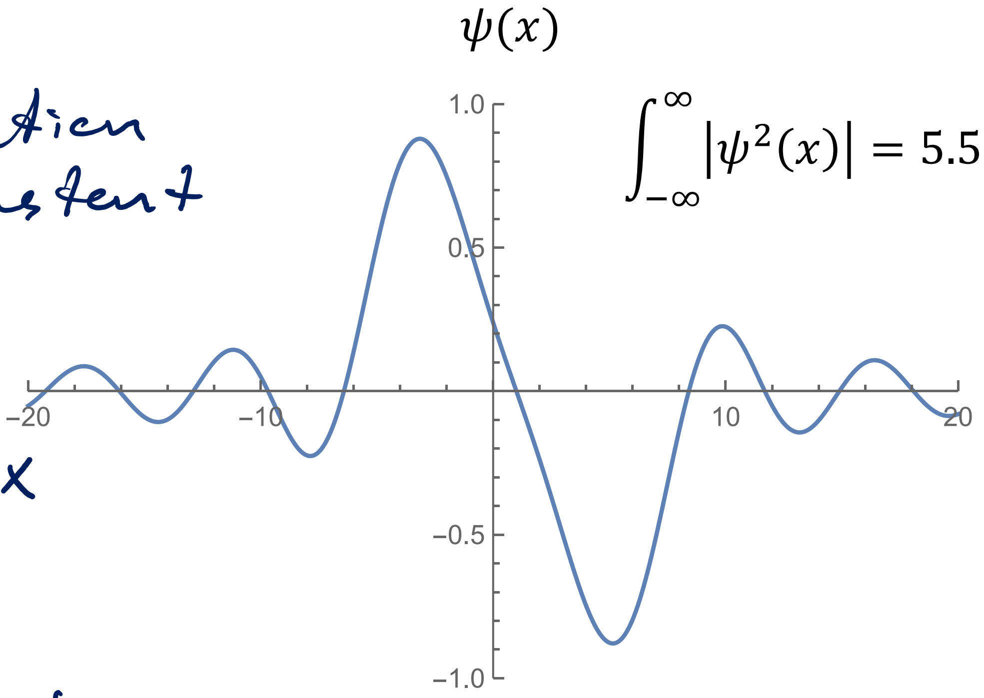
normalization constant

$$P(x) = |k\,\psi(x)|^2$$

$$\int_{-\infty}^{\infty} P(x)\,dx = 1 = \int_{-\infty}^{\infty} |k\,\psi(x)|^2\,dx$$

$$1 = k^2 \int_{-\infty}^{\infty} |\psi(x)|^2\,dx$$

$$k = \sqrt{\frac{1}{\int_{-\infty}^{\infty} |\psi(x)|^2\,dx}}$$



$$\int_{-\infty}^{\infty} |\psi^2(x)| = 5.5$$

Wavefunctions

Quantum particles (electrons, photons, protons, etc.) are described by wavefunctions $\psi(x)$

Wavefunctions follow superposition so can be added $\psi(x) = \psi_1(x) + \psi_2(x)$

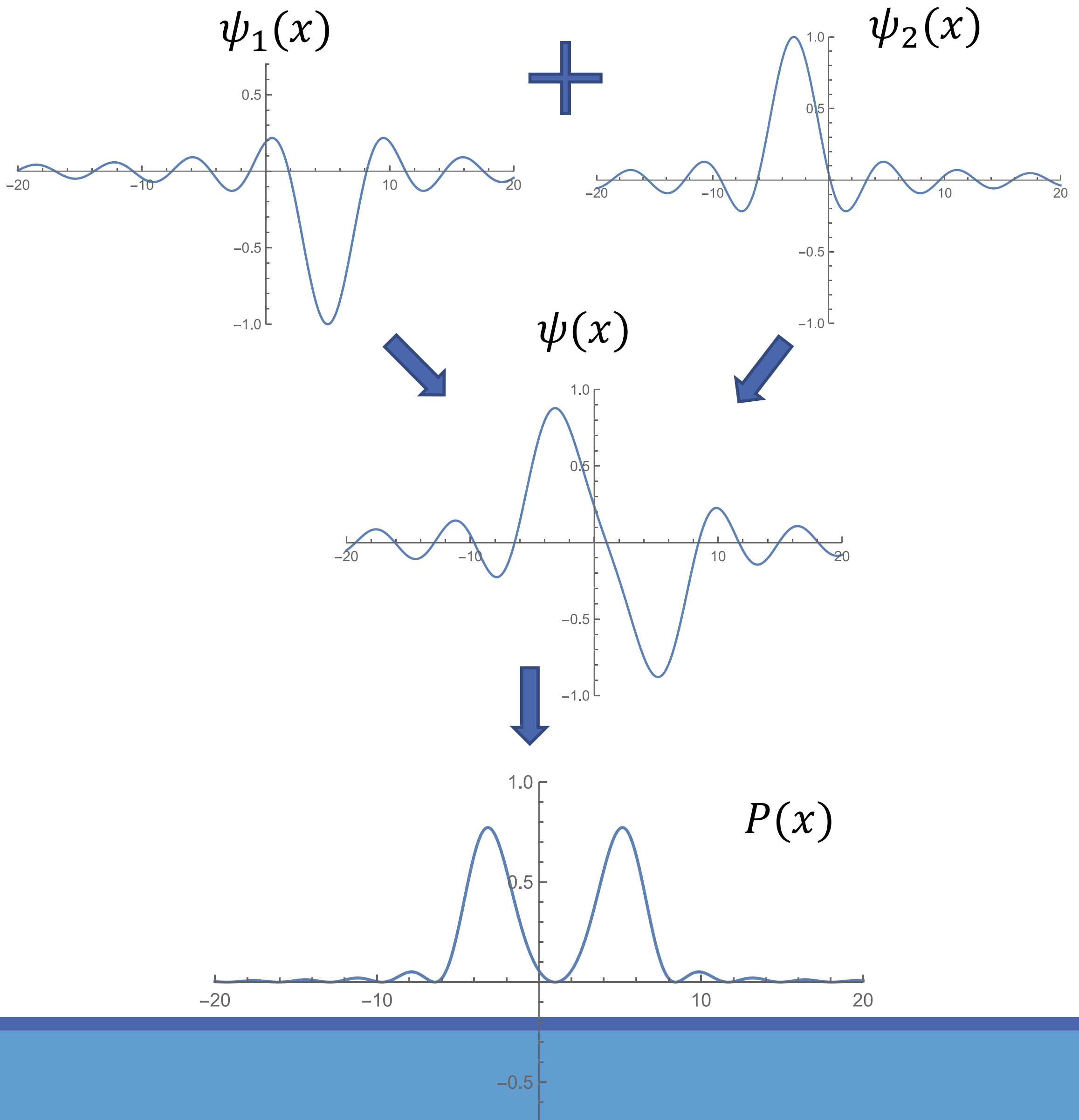
This produces interference effects

The probability density is defined to be

$$P(x) = |\psi(x)|^2$$

Wavefunctions must be normalized

$$\int_{-\infty}^{\infty} |\psi^2(x)| dx = 1$$



Homework Questions

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