Phyx 320
Modern Physics

March 12, 2021

Reading: 39.1 – 39.4

Homework #7 and Reading Reflection Next Thursday 11:59 pm

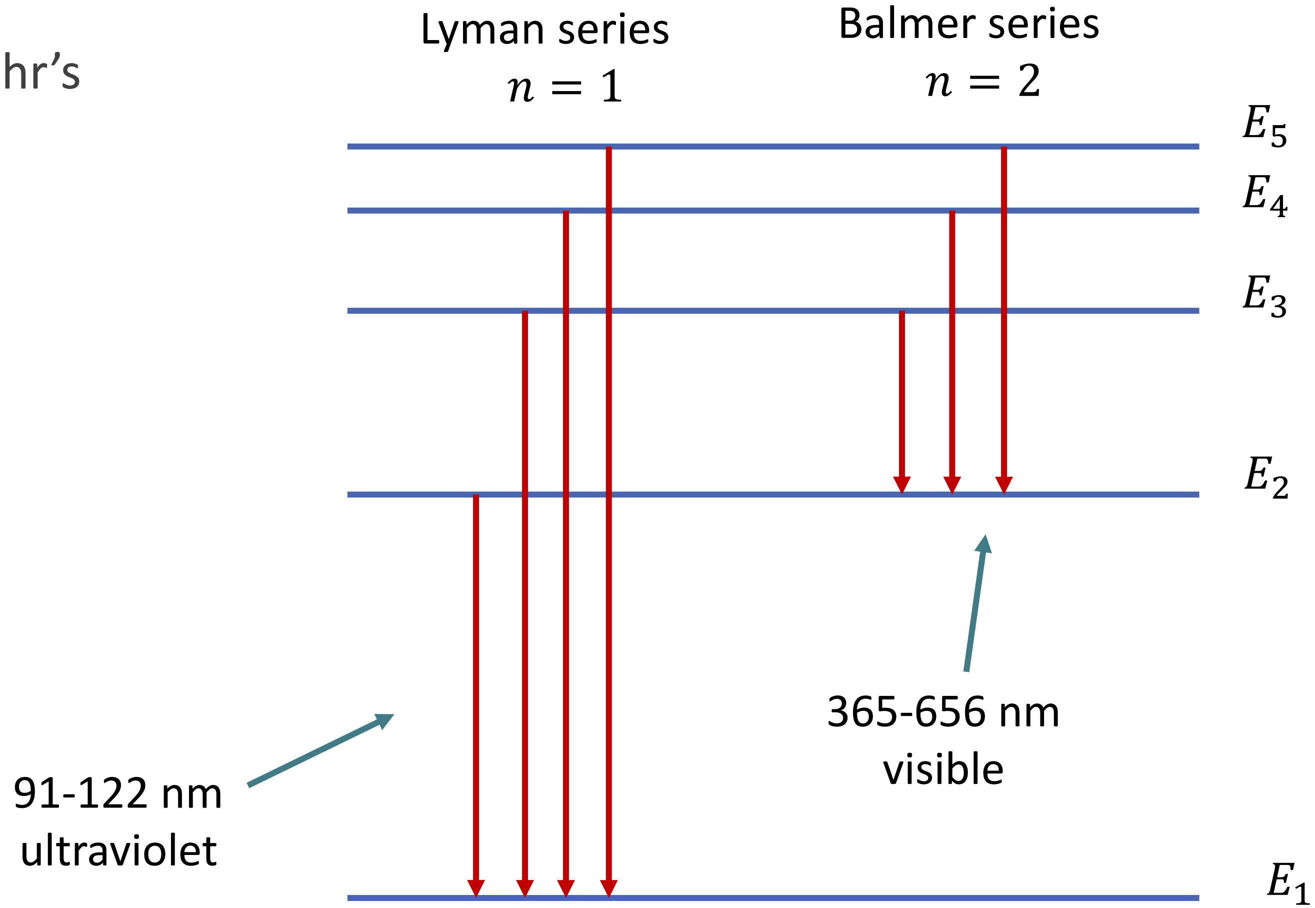
Hydrogen Spectrum

Derived the Balmer formula from Bohr's model of hydrogen:

$$\lambda = \frac{8\pi\epsilon_0 a_B hc}{e^2} \frac{1}{\frac{1}{m^2} - \frac{1}{n^2}}$$

Lyman series final state: $n = 1$

Balmer series final state: $n = 2$



Wave Functions

We used semi-classical theory to describe the hydrogen atom, but we want to build a full theory of quantum mechanics

To do this we need to solidify the wave-particle duality into a structure which we will call a wave function

This wave function will let us calculate the probability of finding a particle in a given state, but we will find that that is **all** we can know

Quantum mechanics does not allow us to know exactly which state each particle will be in, only their probabilities

We will use the double-slit experiment as a launching point since it exhibits both wave and particle properties

Double-Slit Experiment

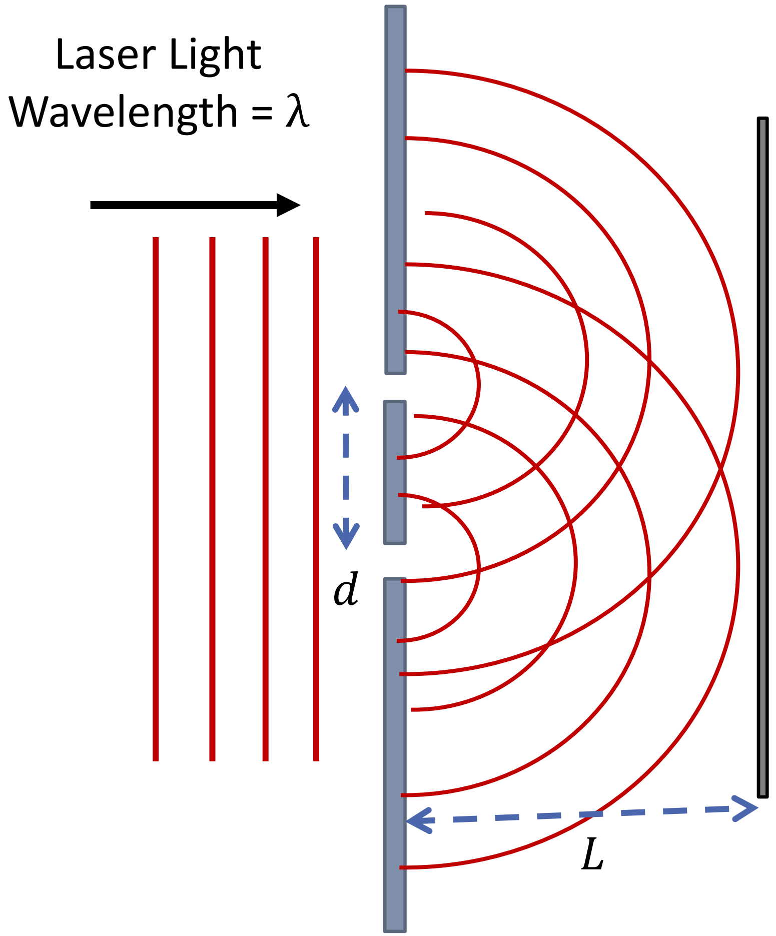
Let's revisit the wave description of double-slit experiment

Electric field waves from each slit follow:

$$A_1 = a\sin(kr_1 - \omega t)$$
$$A_2 = a\sin(kr_2 - \omega t)$$

Total electric wave is the sum of these. Working through the geometry gives:

$$\mathrm{A} = A_1 + A_2 = 2a\cos\left(\frac{\pi\, d\, x}{\lambda L}\right)$$



Double-Slit Experiment

Electric field pattern:

$$A = 2a\cos\left(\frac{\pi\, d\, x}{\lambda L}\right)$$

We don't see the electric field but instead see the intensity:

$$I = A^2 = C\cos^2\left(\frac{\pi d x}{\lambda L}\right)$$

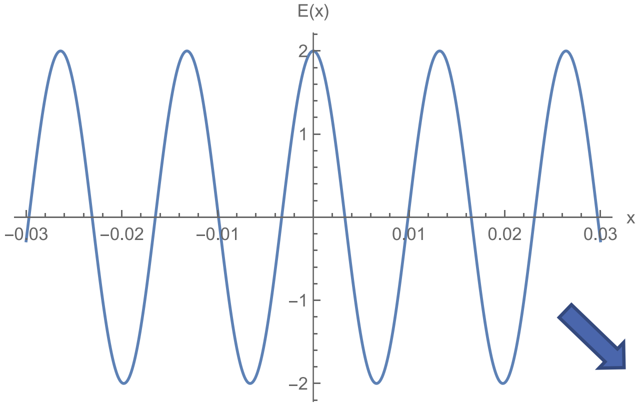
But the photon model tells us that the power of light:

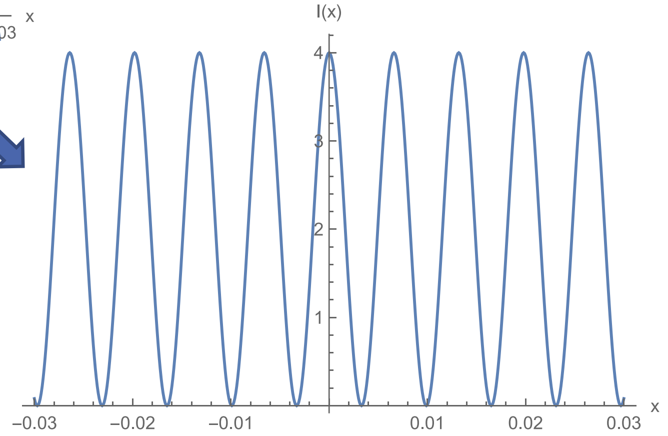
Power

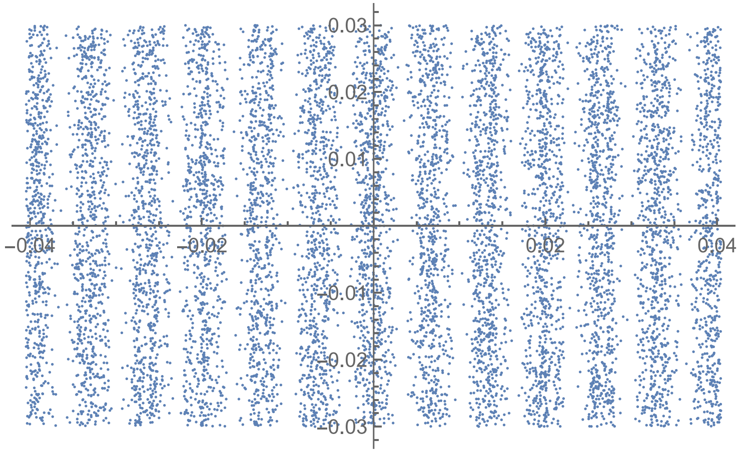
$$P = Rhf$$

Frequency

Rate







Double-Slit Experiment

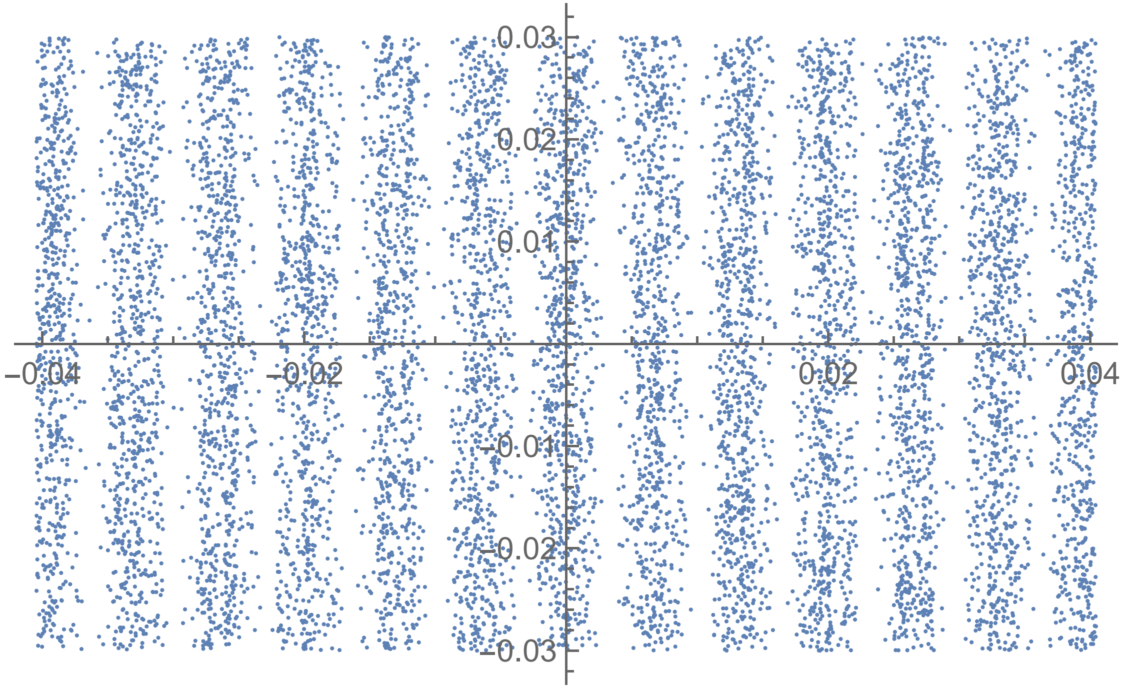
Energy of light:

$$E = Nhf = IA$$

$$I = \frac{Nhf}{A} = C\cos^2\left(\frac{\pi x d}{\lambda L}\right)$$

$$\frac{N}{A} = \frac{C}{hf}\cos^2\left(\frac{\pi x d}{\lambda L}\right)$$

Number photons per area

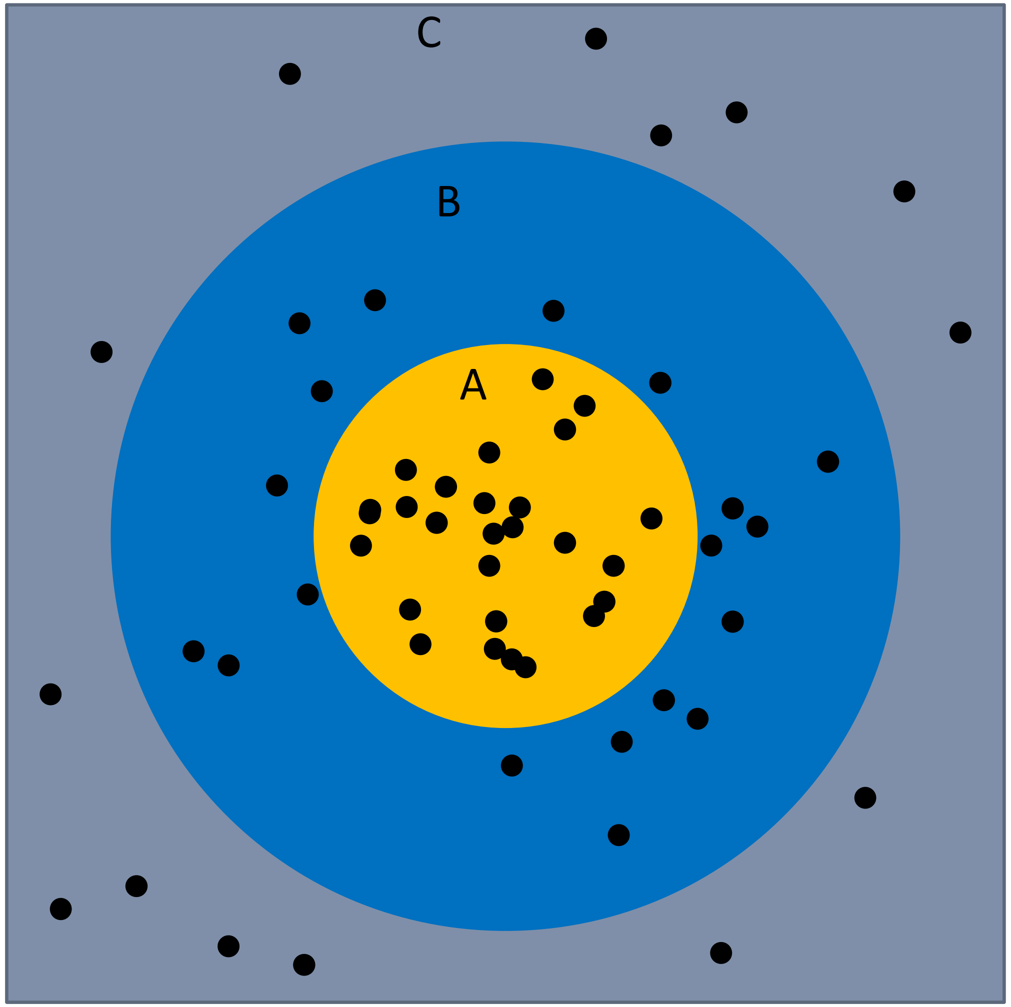


Probability

Let's say we throw N_T darts at a dart board blindfolded

We then measure where they landed and want to predict where the next dart will land

We can't know exact where it will land but can calculate the probability it lands in the different regions



$$P_A = \lim_{N_T \to \infty} \frac{N_A(N_T)}{N_T} \approx \frac{N_A}{N_T}$$

For large N_T

$$P_B = \lim_{N_T \to \infty} \frac{N_B}{N_T}$$

Probability

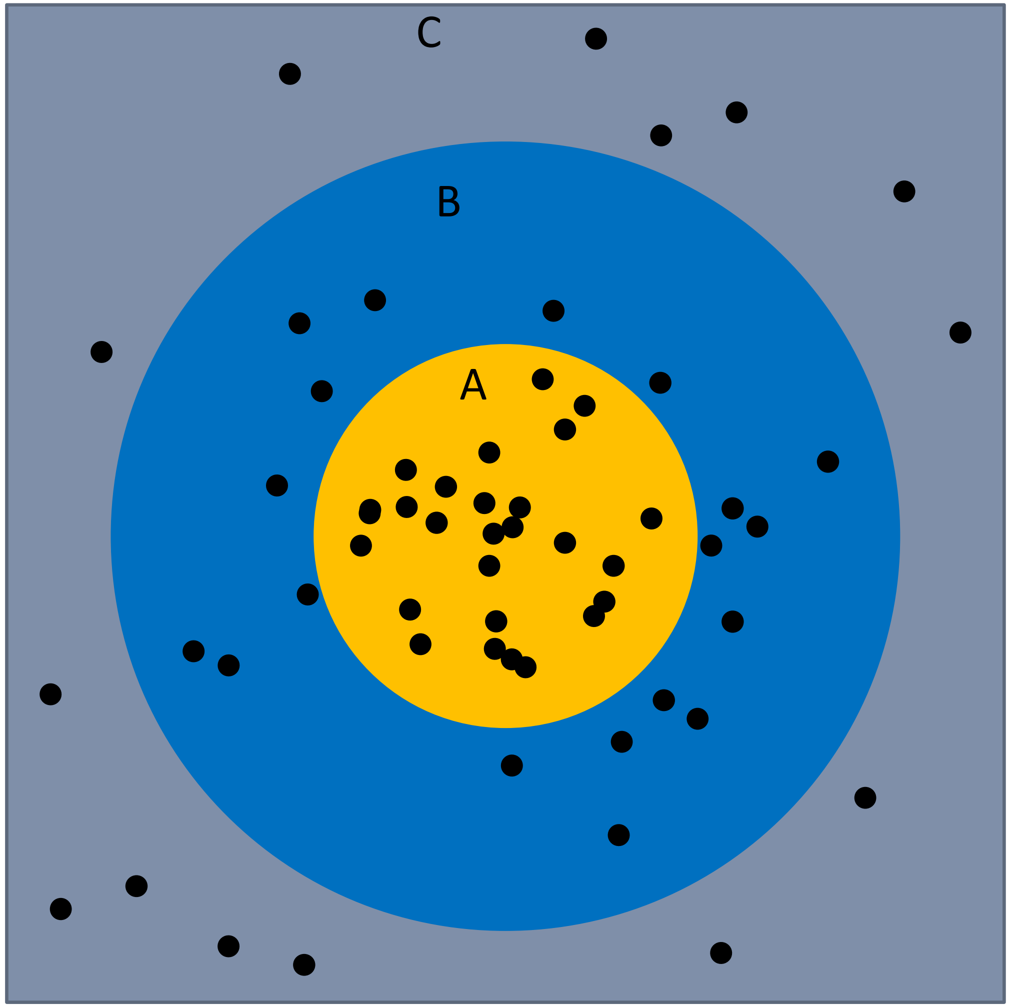
What's the probability of landing in either A or B?

$$P_{A\,or\,B} = \lim_{N_T \to \infty} \frac{N_{A\,or\,B}}{N_T}$$

$$= \lim_{N_T \to \infty} \frac{N_A + N_B}{N_T}$$

$$= \lim_{N_T \to \infty} \frac{N_A}{N_T} + \lim_{N_T \to \infty} \frac{N_B}{N_T}$$

$$P_{A\,or\,B} = P_A + P_B$$

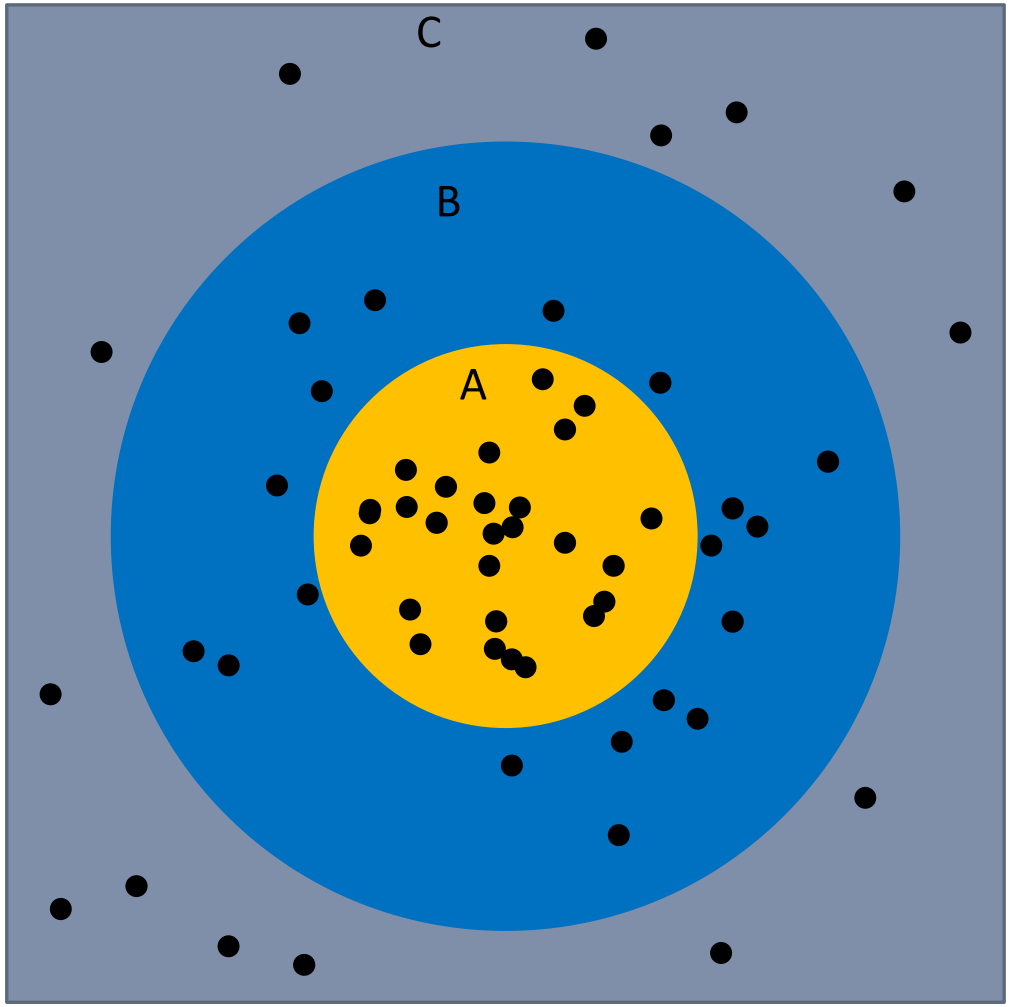


Probability

Once we figure out the probabilities of each region, how many can we expect to find at N more throws?

$$N_{A\ \text{expected}} = N P_A$$

Expected value is the best prediction for the number of darts that will land in region



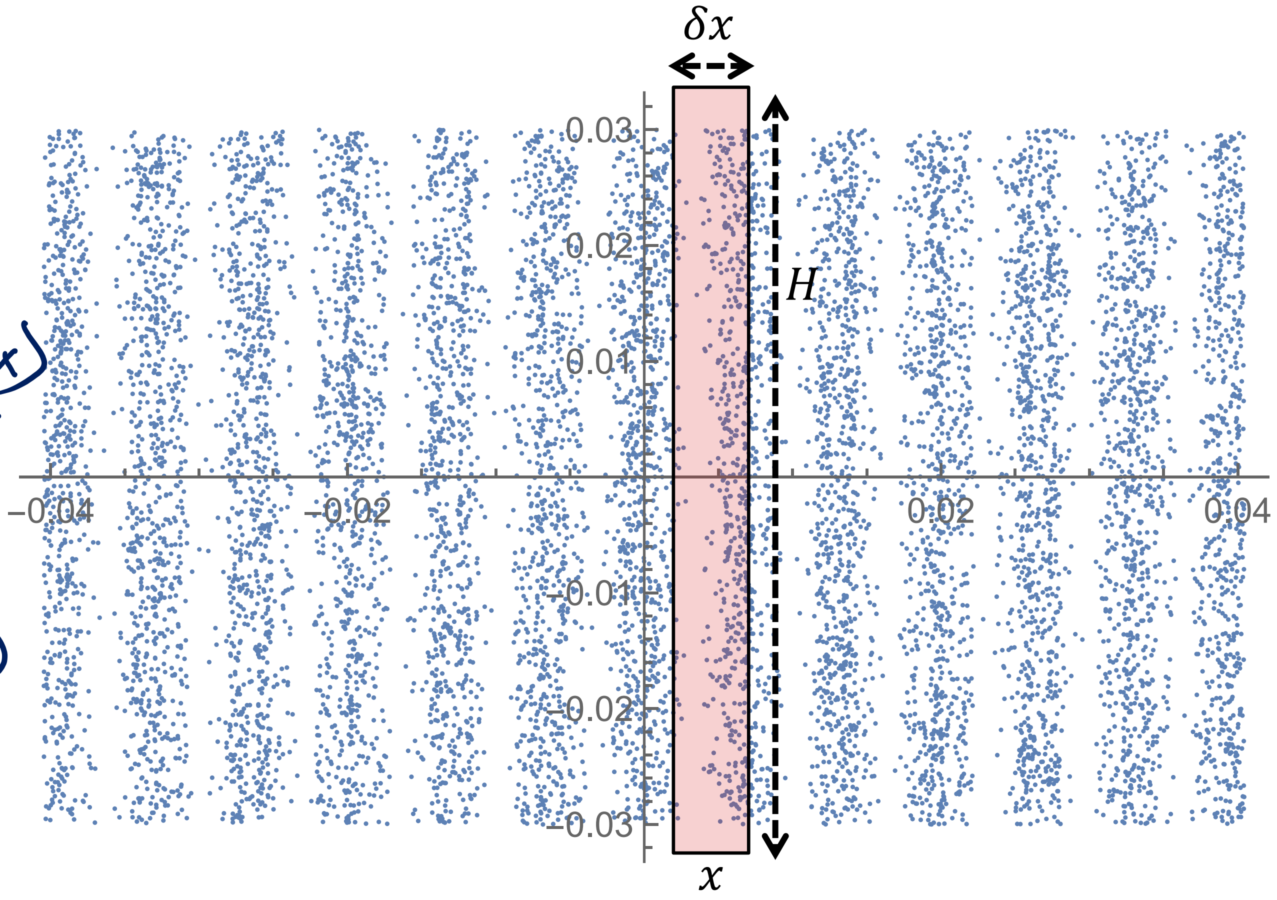
Double-Slit Experiment

Back to double slit, the position of any one photon is **unpredictable**

But we can predict the probabilities and expected values

$$P(\text{in } \delta x \text{ at } x) = \lim_{N_T \to \infty} \frac{N(\text{in } \delta x \text{ at } x)}{N_T}$$

$$N(\text{in } \delta x \text{ at } x) = N \, P(\text{in } \delta x \text{ at } x)$$



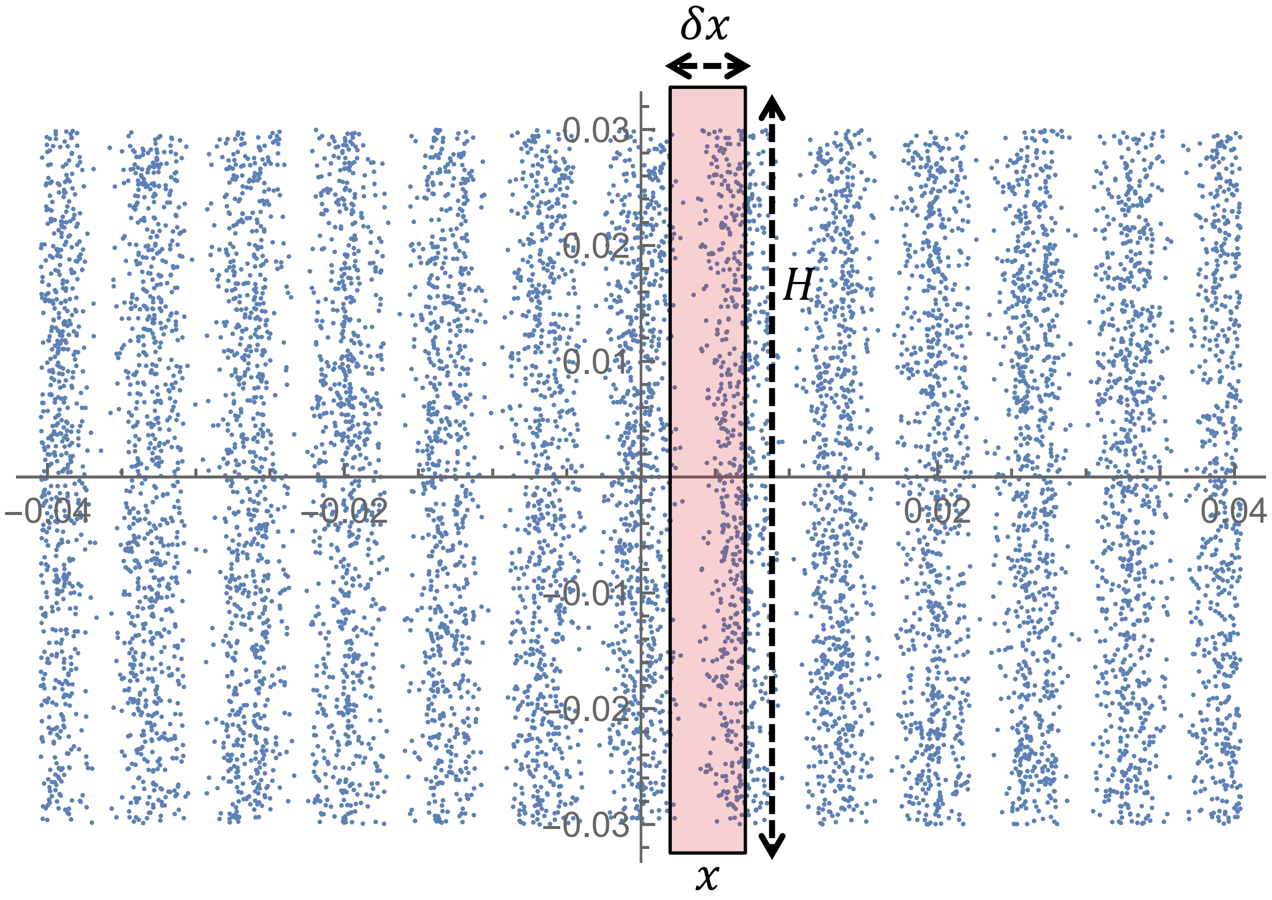
Double-Slit Experiment

Looking at the energy in small strip

$$E(\text{in } \delta x \text{ at } x) = I(x) A = I(x) H \, \delta x$$

$$E(\text{in } \delta x \text{ at } x) = N(\text{in } \delta x \text{ at } x) \, hF$$

$$N(\text{in } \delta x \text{ at } x) = \frac{H}{hF} I(x) \, \delta x$$



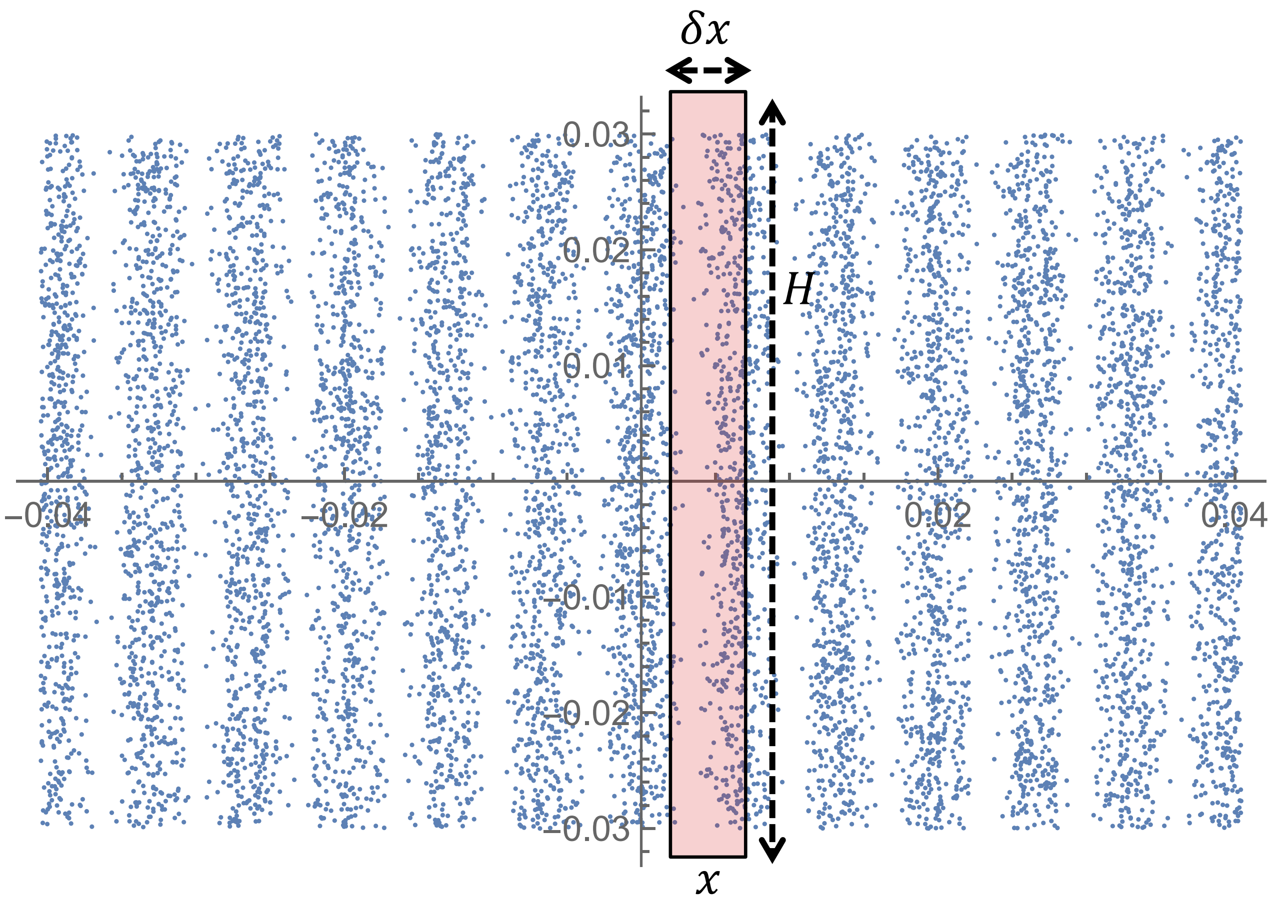
Double-Slit Experiment

Let's tie this back to probability

$$P(\text{in } \delta x \text{ at } x) = \frac{N(\text{in } \delta x \text{ at } x)}{N_T}$$

$$= \frac{H}{hfN_T} I(x)\,\delta x$$

$$= \frac{H}{hfN_T} \left(A(x)\right)^2 \delta x$$



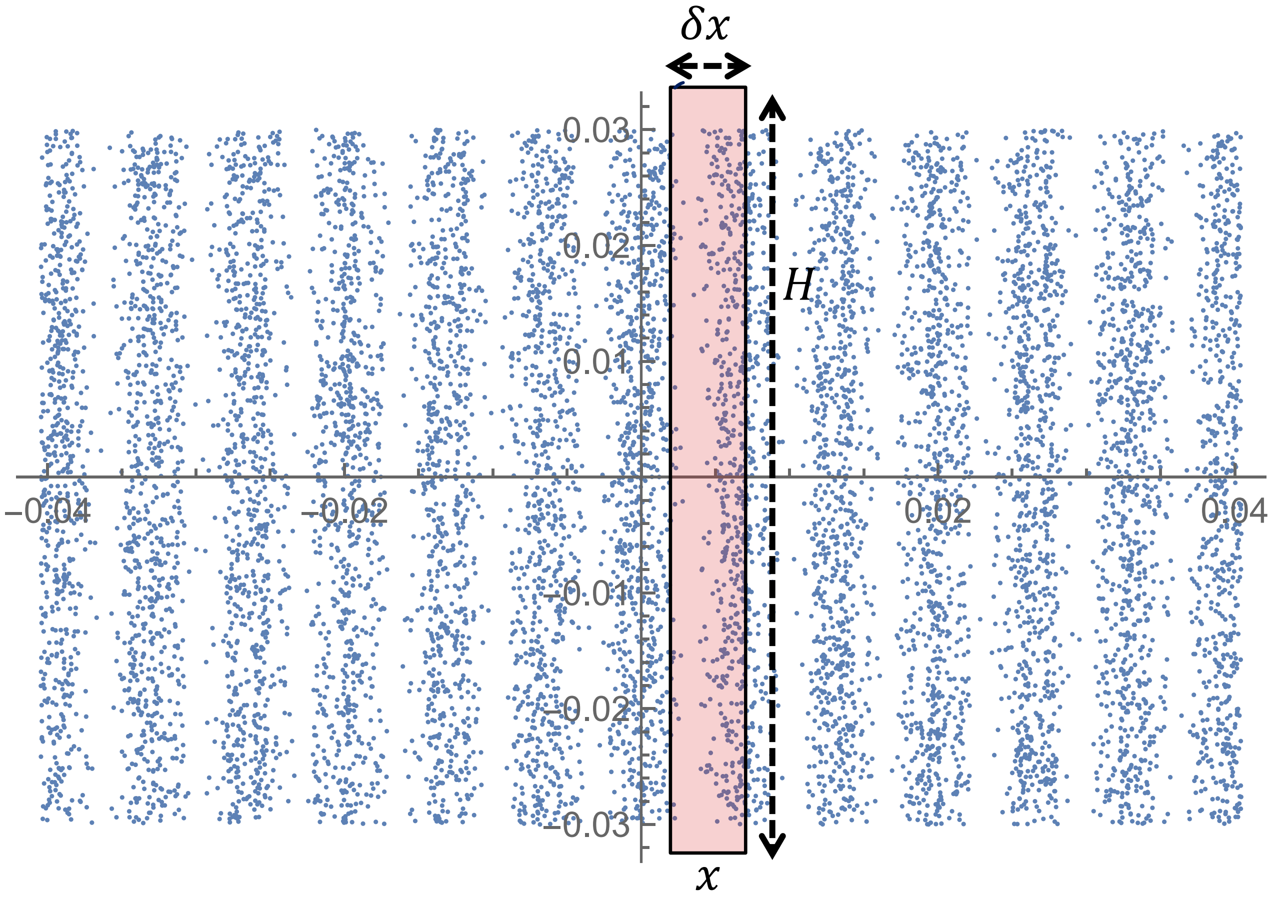
Probability Density

We define a probability per unit distance (probability density):

$$Prob(in\ \delta x\ at\ x) = P(x)\delta x$$

$$P(x) = \frac{H}{4FN_T}\left(A(x)\right)^2$$

$$= \frac{H}{4FN_T}\cos^2\left(\frac{\pi x d}{\lambda L}\right)$$



Probability Density

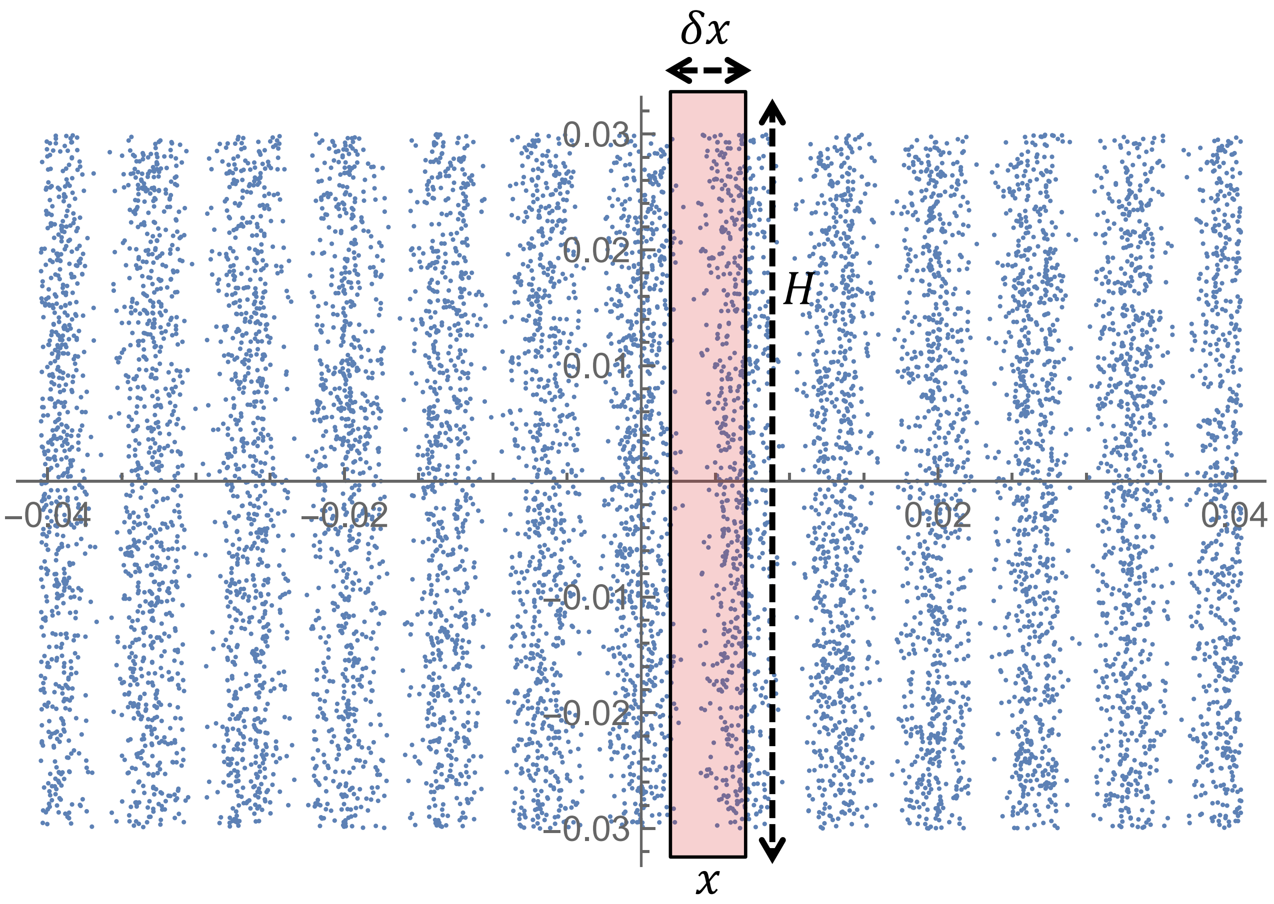
The probability density for photons is directly proportional to the square of the amplitude of the corresponding electromagnetic wave

$$P(x) \propto A(x)^2 \cancel{\propto} I(x)$$

Next lecture we will discuss the wavefunction which is generalization of this to every quantum particle

$$P(x) \propto |\psi(x)|^2$$

Wavefunction
(complex number)



Quiz 6

1. Neon is the tenth element on the periodic table (Z=10). If we had a neon atom with only one electron, what wavelength would be emitted when the electron transitions from the m=3 state to the n=2 state (using the Bohr model of the atom)?

2. Derive the relativistic version of the de Broglie wavelength. What is the wavelength of an electron (m_e=9.11 × 10^-31 kg) travelling at v=0.9c?

1. $Z = 10, \ m = 3, \ n = 2$

$$\lambda_0 = \frac{91.18\,\text{nm}}{Z^2} = 0.9118\,\text{nm}$$

$$\lambda = \frac{\lambda_0}{\frac{1}{n^2} - \frac{1}{m^2}} = 6.565\,\text{nm}$$

X-ray

Quiz 6

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2. Derive the relativistic version of the de Broglie wavelength. What is the wavelength of an electron (m_e=9.11 × 10^-31 kg) travelling at v=0.9c?

2.

$$\lambda = \frac{h}{p} = \frac{h\sqrt{1-(v/c)^2}}{mv}$$

$$p = \gamma m v = \frac{mv}{\sqrt{1-(v/c)^2}}$$

$$\lambda = 1.175 \times 10^{-12}\ m$$

Homework Questions

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