Phyx 320
Modern Physics

March 5, 2021

Reading: 38.5 – 38.7

Homework #6 and Reading Reflection Next Tuesday 11:59 pm

Bohr Hydrogen

Radii of electron orbit in hydrogen is quantized

$$r_n = a_B n^2 \qquad n = 1, 2, 3 \ldots$$

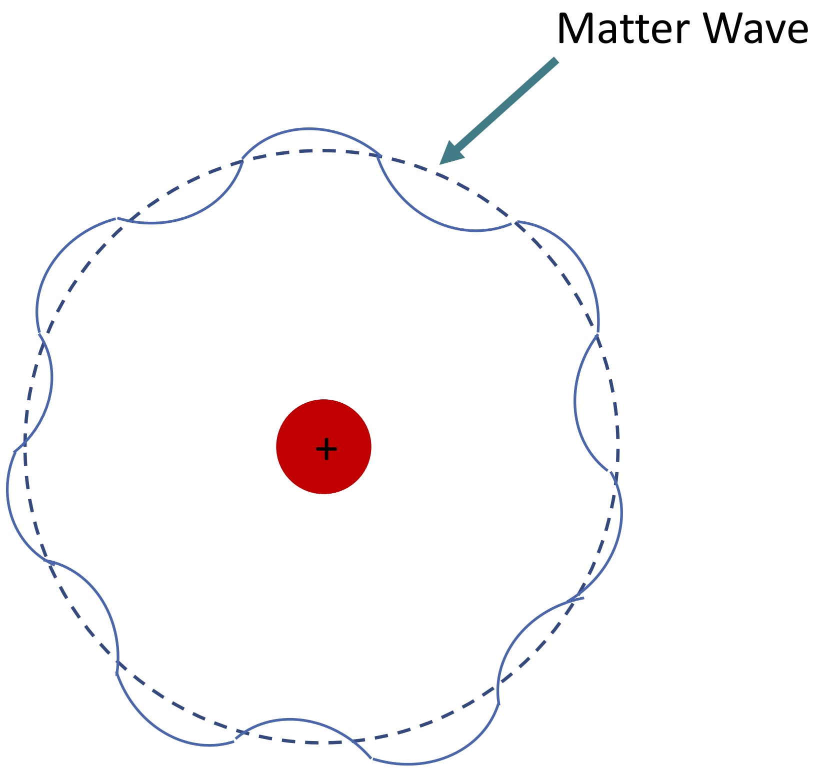
Defined Bohr radius

$$a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 0.0529\,nm$$

Examples of electron radius:

$$r_1 = 1a_B = 0.053\,nm$$
$$r_2 = 4a_B = 0.212\,nm$$
$$r_3 = 9a_B = 0.476\,nm$$

Hydrogen atoms at other radii **can not** exist



Bohr Hydrogen

Let’s derive the energy levels of hydrogen

Bohr Hydrogen

Let's derive the energy levels of hydrogen

$$r = a_B n^2 \quad (n = 1, 2, 3 \ldots)$$

$$v = \frac{n\hbar}{mr} = \frac{\hbar}{m a_B}\frac{1}{n}$$

$$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= \frac{1}{2}m\left(\frac{\hbar}{m a_B}\frac{1}{n}\right)^2 - \frac{e^2}{4\pi\varepsilon_0 a_B n^2}$$

$$= -\frac{1}{n^2}\left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{2a_B}\right)$$

13.60 eV

$= 2.18 \times 10^{-18}$ J

Bohr Hydrogen

Can describe hydrogen states by one quantum number: n

Energy follows $\sim 1/n^2$

Radius follows $\sim n^2$

Each n corresponds to a unique energy and radius

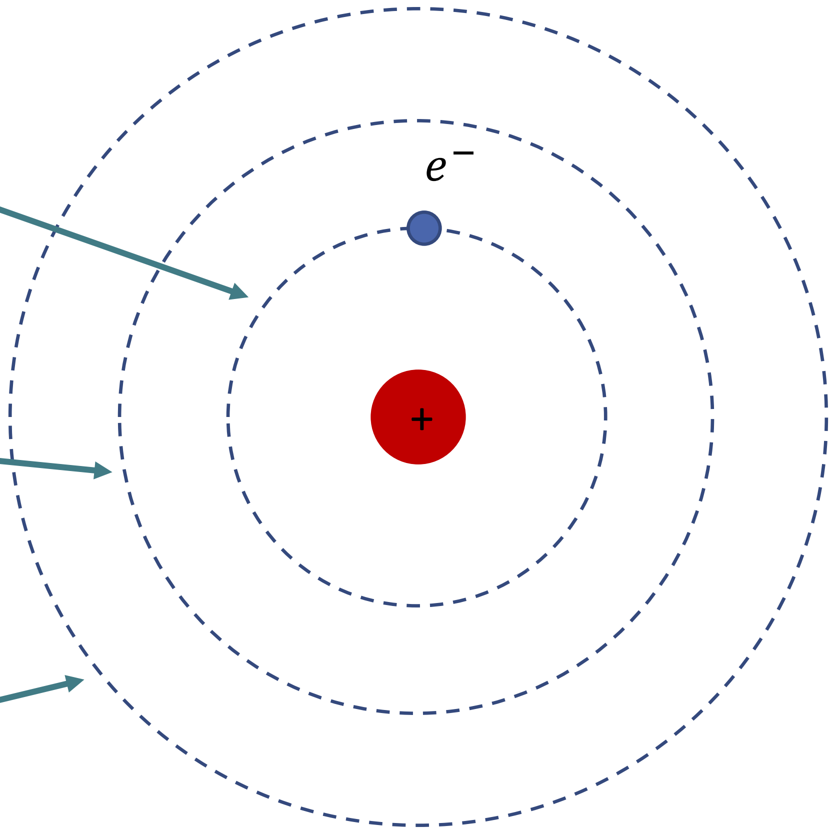
Energy is negative since the electric potential energy is only zero when $r \to \infty$

Requires energy to pull the electron away from the proton

$n = 1$
$E_1 = -13.60\ eV$
$r_1 = 0.053\ nm$

$n = 2$
$E_1 = -3.40\ eV$
$r_1 = 0.212\ nm$

$n = 3$
$E_1 = -1.51\ eV$
$r_1 = 0.476\ nm$



$$U_e = \frac{q_1 q_2}{4\pi \varepsilon_0 r}$$

Bohr Hydrogen

What about angular momentum?

$L = mvr$

de Broglie:

$$\left(2\pi r = n\lambda = n\frac{h}{mv}\right) mv$$

$$2\pi mvr = nh$$

$$L = mvr = n\frac{h}{2\pi} = n\hbar$$

$n = 1, 2, 3 \ldots$

quantized

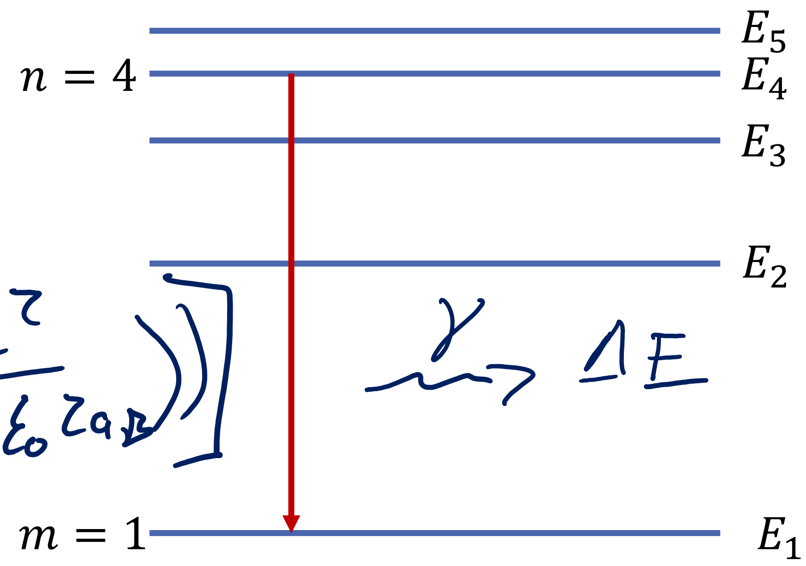
quantum angular momentum

Hydrogen Spectrum

Does this model describe the hydrogen spectrum?

$$F = \frac{E_i - E_f}{h} = \frac{E_n - E_m}{h}$$

$$F = \frac{1}{h}\left[-\frac{1}{n^2}\left(\frac{e^2}{4\pi\varepsilon_0 2a_B}\right) - \left(-\frac{1}{m^2}\left(\frac{e^2}{4\pi\varepsilon_0 2a_B}\right)\right)\right]$$



$\gamma \rightsquigarrow \Delta E$

$$f = \frac{e^2}{8\pi h \varepsilon_0 a_B}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

91.12 nm

$$\lambda = \frac{c}{f} = \frac{8\pi h c \varepsilon_0 a_B}{e^2} \frac{1}{\frac{1}{m^2} - \frac{1}{n^2}}$$

← Balmer Formula

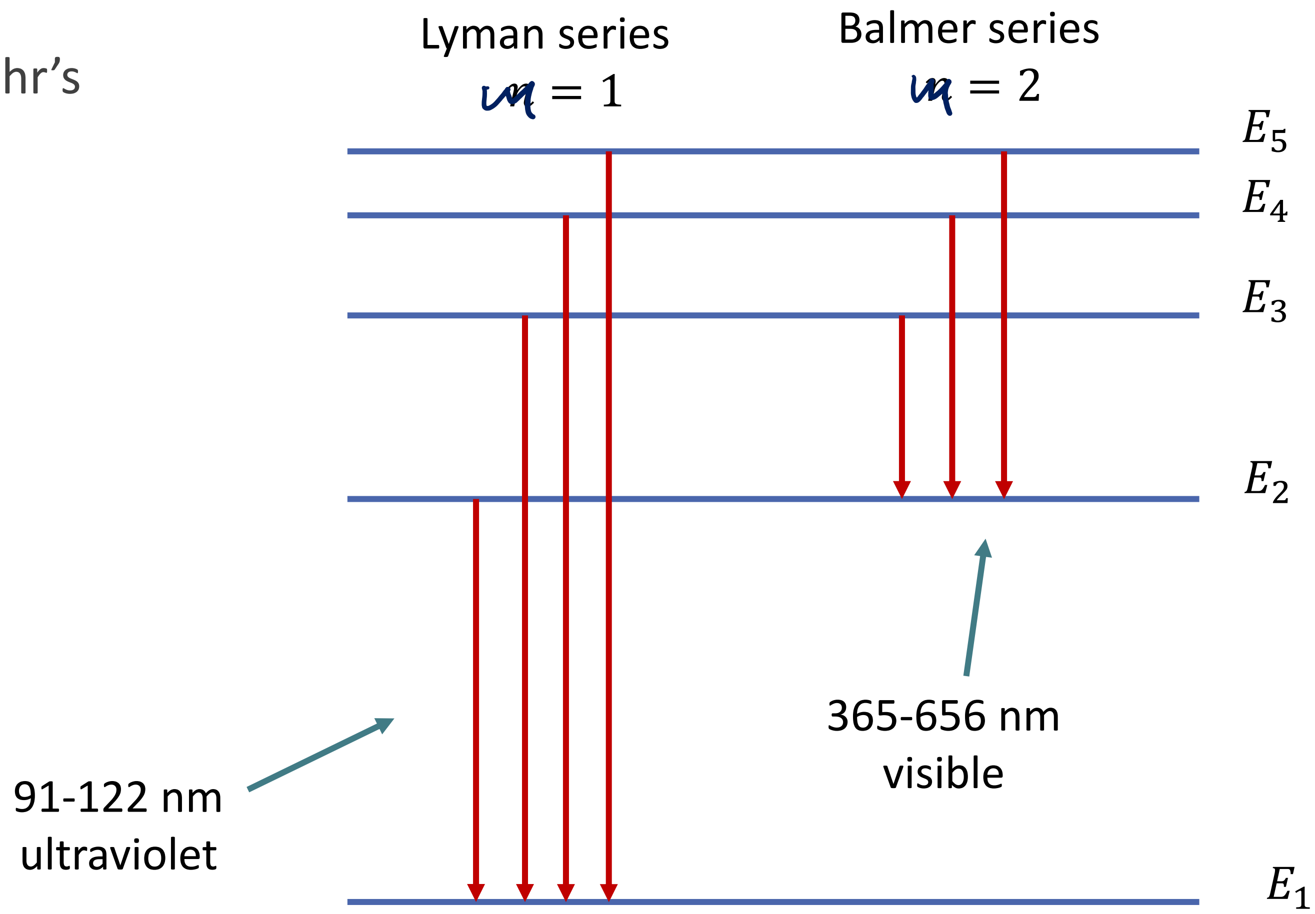
Hydrogen Spectrum

Derived the Balmer formula from Bohr's model of hydrogen:

$$\lambda = \frac{8\pi\epsilon_0 a_B hc}{e^2}\frac{1}{\frac{1}{m^2}-\frac{1}{n^2}}$$

Lyman series final state: $m = 1$

Balmer series final state: $m = 2$



Hydrogen Like Atoms

Bohr model can be used for other elements as long as they have only one electron

Atomic number = number of protons

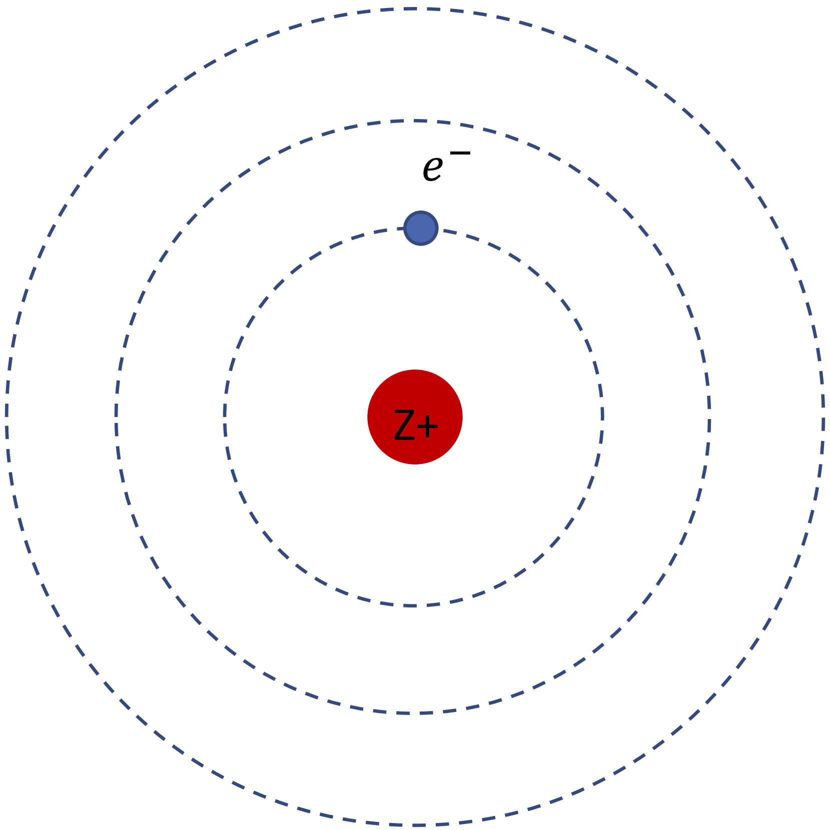
$$U_e = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

Shifts all equations that we've derived

Energy and emission spectrum:

$$E = -13.60\ eV\frac{Z^2}{n^2}$$

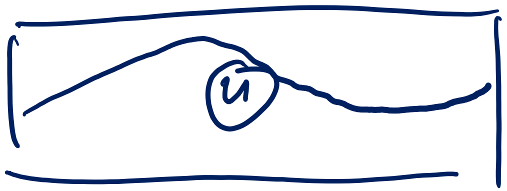
$$\lambda_0 = \frac{91.18\ nm}{Z^2}$$



Quiz 5

1. What is the de Broglie wavelength for a neutron ($m_n = 1.675 \times 10^{-27}\ kg$) traveling at 10 m/s?

2. What is the ground state energy of a neutron in a one-dimensional box with a length of 1 angstrom ($10^{-11}\ m$)?



1. $\lambda = \frac{h}{p} = \frac{h}{mv}$

$\lambda = 3.96 \times 10^{-8}\ m$

$= 39.6\ nm$

2. $E_1 = \frac{h^2}{8mL^2}$ $L = 1\ Å = 10^{-11}\ m$

$= 3.28 \times 10^{-21}\ J$

$= 20.45\ meV$

Quiz 5

1. What is the de Broglie wavelength for a neutron ($m_n \ = \ 1.675 \times 10^{-27} \ kg$) traveling at 10 m/s?

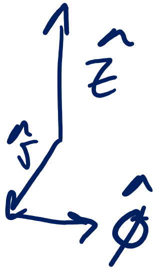
2. What is the ground state energy of a neutron in a one-dimensional box with a length of 1 angstrom ($10^{-11} \ m$)?

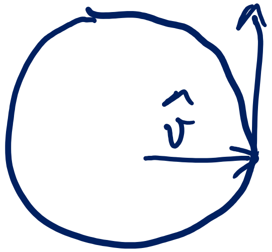
Homework Questions

$$\vec{L} = \vec{r} \times \vec{p}$$

$$E_2 \Rightarrow n = 2$$

$$L_z = 2\hbar$$





$$\vec{p} = mv\hat{\phi}$$

$$\vec{r} = r\hat{r}$$

$$\vec{L} = r\hat{r} \times mv\hat{\phi} = rmv\,\hat{z}$$

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